

The Central Limit Theorem I — the miracle, watched live

Lesson 70 · Module 5 — Limit Theorems · Probability & Statistics · dailymaths.uk/probstat

Practice problems

1. A fair six-sided die is rolled $n = 40$ times. Using the population mean $\mu = 3.5$ and variance $\sigma^2 = 2.9167$, find the probability that the sample mean $\bar{X}$ is between 3.3 and 3.7.
2. A population has a mean $\mu = 100$ and a standard deviation $\sigma = 20$. If a sample of $n = 64$ is taken, what is the probability that the sample mean $\bar{X}$ is greater than 105?
3. The number of emails received per hour follows an exponential distribution with a mean of $\mu = 2$ emails per hour. If we monitor the process for $n = 100$ hours, what is the probability that the average number of emails per hour is less than 1.8?

Solutions

1. A fair six-sided die is rolled $n = 40$ times. Using the population mean $\mu = 3.5$ and variance $\sigma^2 = 2.9167$, find the probability that the sample mean $\bar{X}$ is between 3.3 and 3.7.

- Calculate the standard error: $SE = \sqrt{2.9167/40} \approx 0.2700$.
- Find the z-score for 3.3: $z_1 = (3.3 - 3.5)/0.2700 \approx -0.74$.
- Find the z-score for 3.7: $z_2 = (3.7 - 3.5)/0.2700 \approx 0.74$.
- Find the area between $z = -0.74$ and $z = 0.74$ using the normal table: $\Phi(0.74) - \Phi(-0.74) \approx 0.7704 - 0.2296 = 0.5408$.

Answer: 0.5408

2. A population has a mean $\mu = 100$ and a standard deviation $\sigma = 20$. If a sample of $n = 64$ is taken, what is the probability that the sample mean $\bar{X}$ is greater than 105?

- Calculate the standard error: $SE = 20/\sqrt{64} = 20/8 = 2.5$.
- Calculate the z-score: $z = (105 - 100)/2.5 = 5/2.5 = 2.0$.
- Find the probability $P(Z > 2.0) = 1 - \Phi(2.0) \approx 1 - 0.9772 = 0.0228$.

Answer: 0.0228

3. The number of emails received per hour follows an exponential distribution with a mean of $\mu = 2$ emails per hour. If we monitor the process for $n = 100$ hours, what is the probability that the average number of emails per hour is less than 1.8?

- For an exponential distribution, the mean $\mu = 1/\lambda$ and the variance $\sigma^2 = 1/\lambda^2$. Since $\mu = 2$, then $\sigma^2 = 2^2 = 4$.
- Calculate the standard error: $SE = \sqrt{4/100} = 2/10 = 0.2$.
- Calculate the z-score: $z = (1.8 - 2.0)/0.2 = -0.2/0.2 = -1.0$.
- Find the probability $P(Z < -1.0) = \Phi(-1.0) \approx 0.1587$.

Answer: 0.1587