

Markov and Chebyshev Inequalities

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Practice problems

1. A non-negative random variable X has an expected value $E[X] = 12$. Use Markov's Inequality to find the maximum possible probability that X is at least 36.
2. A random variable Y has a mean $\mu = 50$ and a variance $\sigma^2 = 16$. What is the maximum probability that Y falls outside the interval $[38, 62]$?
3. A population has a mean μ and standard deviation σ . If we take a sample of size $n = 400$, what is the maximum probability that the sample mean $\bar{X}$ differs from μ by more than 0.05σ ?

Solutions

1. A non-negative random variable X has an expected value $E[X] = 12$. Use Markov's Inequality to find the maximum possible probability that X is at least 36.

- Identify the knowns: $E[X] = 12$ and $a = 36$.
- Apply Markov's Inequality: $P(X \geq a) \leq \frac{E[X]}{a}$.
- Substitute the values: $P(X \geq 36) \leq \frac{12}{36}$.
- Simplify the fraction: $\frac{12}{36} = \frac{1}{3}$.

Answer: $1/3$

2. A random variable Y has a mean $\mu = 50$ and a variance $\sigma^2 = 16$. What is the maximum probability that Y falls outside the interval $[38, 62]$?

- Find the standard deviation: $\sigma = \sqrt{16} = 4$.
- Determine the distance from the mean to the boundary: $62 - 50 = 12$ (or $50 - 38 = 12$).
- Calculate k , the number of standard deviations: $k = \frac{12}{4} = 3$.
- Apply Chebyshev's Inequality: $P(|Y - 50| \geq 12) \leq \frac{1}{3^2}$.
- Compute the result: $\frac{1}{9} \approx 0.1111$.

Answer: $1/9$

3. A population has a mean μ and standard deviation σ . If we take a sample of size $n = 400$, what is the maximum probability that the sample mean $\bar{X}$ differs from μ by more than 0.05σ ?

- Recall the standard error of the mean: $SE = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{400}} = \frac{\sigma}{20} = 0.05\sigma$.
- The distance we are interested in is $\epsilon = 0.05\sigma$.
- Calculate k by dividing the distance by the standard error: $k = \frac{0.05\sigma}{0.05\sigma} = 1$.
- Apply Chebyshev's Inequality: $P(|\bar{X} - \mu| \geq 0.05\sigma) \leq \frac{1}{1^2}$.
- The result is 1, which means the probability is at most 100%. (Note: This shows that for this specific sample size and distance, the universal bound is not restrictive).

Answer: 1