

The sample mean as a random variable

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Practice problems

1. A population has a mean $\mu = 80$ and a standard deviation $\sigma = 12$. If a random sample of size $n = 36$ is taken, what is the standard error of the sample mean $\bar{X}$?
2. The variance of a population is $\sigma^2 = 400$. If the sample mean $\bar{X}$ has a variance of 4, what was the sample size n used?
3. A researcher wants to estimate the average height of a plant species. The population standard deviation is $\sigma = 5$ cm. They want the standard error of the sample mean to be no more than 0.5 cm. What is the minimum sample size n required?

Solutions

1. A population has a mean $\mu = 80$ and a standard deviation $\sigma = 12$. If a random sample of size $n = 36$ is taken, what is the standard error of the sample mean $\bar{X}$?

- Identify the given values: $\sigma = 12$ and $n = 36$.
- Use the formula for standard error: $\text{SE}_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$.
- Substitute the values: $\text{SE}_{\bar{X}} = \frac{12}{\sqrt{36}}$.
- Calculate the square root: $\sqrt{36} = 6$.
- Divide: $12/6 = 2$.

Answer: 2

2. The variance of a population is $\sigma^2 = 400$. If the sample mean $\bar{X}$ has a variance of 4, what was the sample size n used?

- Use the formula for the variance of the sample mean: $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$.
- Substitute the known values: $4 = \frac{400}{n}$.
- Multiply both sides by n : $4n = 400$.
- Divide by 4: $n = 100$.

Answer: 100

3. A researcher wants to estimate the average height of a plant species. The population standard deviation is $\sigma = 5$ cm. They want the standard error of the sample mean to be no more than 0.5 cm. What is the minimum sample size n required?

- Set up the inequality for standard error: $\frac{\sigma}{\sqrt{n}} \leq 0.5$.
- Substitute $\sigma = 5$: $\frac{5}{\sqrt{n}} \leq 0.5$.
- Rearrange to solve for $\sqrt{n}$: $\sqrt{n} \geq \frac{5}{0.5}$.
- Simplify the fraction: $\sqrt{n} \geq 10$.
- Square both sides: $n \geq 100$.

Answer: 100