

Covariance clinic — portfolio variance

Lesson 65 · Module 4 — Joint Distributions & Dependence · Probability & Statistics · daily-maths.uk/probstat

Practice problems

1. A portfolio consists of two assets with weights $w_X = 0.6$ and $w_Y = 0.4$. The standard deviations are $\sigma_X = 0.15$ and $\sigma_Y = 0.25$, and the correlation is $\rho = 0.3$. Calculate the portfolio variance $\text{Var}(R)$.
2. Two assets have equal weights $w = 0.5$. Both have a standard deviation of 0.2. If the assets are perfectly negatively correlated ($\rho = -1$), what is the portfolio standard deviation σ_P ?
3. An investor wants to combine a risky asset X ($\sigma_X = 0.4$) with a risk-free asset Y ($\sigma_Y = 0$). If they put 30% of their money in asset X , what is the portfolio standard deviation σ_P ?

Solutions

1. A portfolio consists of two assets with weights $w_X = 0.6$ and $w_Y = 0.4$. The standard deviations are $\sigma_X = 0.15$ and $\sigma_Y = 0.25$, and the correlation is $\rho = 0.3$. Calculate the portfolio variance $\text{Var}(R)$.

- Identify values: $w = 0.6, \sigma_X = 0.15, \sigma_Y = 0.25, \rho = 0.3$.
- Compute first term: $0.6^2 \times 0.15^2 = 0.36 \times 0.0225 = 0.0081$.
- Compute second term: $0.4^2 \times 0.25^2 = 0.16 \times 0.0625 = 0.01$.
- Compute interaction term: $2 \times 0.6 \times 0.4 \times 0.3 \times 0.15 \times 0.25 = 0.48 \times 0.3 \times 0.0375 = 0.0054$.
- Sum the terms: $0.0081 + 0.01 + 0.0054 = 0.0235$.

Answer: 0.0235

2. Two assets have equal weights $w = 0.5$. Both have a standard deviation of 0.2. If the assets are perfectly negatively correlated ($\rho = -1$), what is the portfolio standard deviation σ_P ?

- Identify values: $w = 0.5, \sigma_X = 0.2, \sigma_Y = 0.2, \rho = -1$.
- Compute variance: $\text{Var}(R) = 0.5^2(0.2^2) + 0.5^2(0.2^2) + 2(0.5)(0.5)(-1)(0.2)(0.2)$.
- Simplify: $0.25(0.04) + 0.25(0.04) - 0.5(0.04) = 0.01 + 0.01 - 0.02 = 0$.
- The standard deviation is $\sqrt{0} = 0$.

Answer: 0

3. An investor wants to combine a risky asset X ($\sigma_X = 0.4$) with a risk-free asset Y ($\sigma_Y = 0$). If they put 30% of their money in asset X , what is the portfolio standard deviation σ_P ?

- Identify values: $w = 0.3, \sigma_X = 0.4, \sigma_Y = 0, \rho = 0$ (since Y is constant).
- Apply formula: $\text{Var}(R) = 0.3^2(0.4^2) + 0.7^2(0^2) + 2(0.3)(0.7)(0)(0.4)(0)$.
- Simplify: $\text{Var}(R) = 0.09 \times 0.16 = 0.0144$.
- Compute standard deviation: $\sigma_P = \sqrt{0.0144} = 0.12$.

Answer: 0.12