

The Law of Iterated Expectation

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Practice problems

1. Let X be a random variable such that $P(X = 1) = 0.6$ and $P(X = 2) = 0.4$. Given X , Y follows a binomial distribution with parameters $n = X$ and $p = 0.4$. Find $E[Y]$.
2. Suppose X is uniformly distributed on the interval $[0, 10]$. Given $X = x$, Y is uniformly distributed on the interval $[0, x]$. Find $E[Y]$.
3. Let X follow a Poisson distribution with $\lambda = 3$. Given X , Y follows a Poisson distribution with parameter $\lambda = X$. Find $E[Y]$.

Solutions

1. Let X be a random variable such that $P(X = 1) = 0.6$ and $P(X = 2) = 0.4$. Given X , Y follows a binomial distribution with parameters $n = X$ and $p = 0.4$. Find $E[Y]$.
 - First, find the conditional expectation $E[Y | X]$. For a binomial distribution, the mean is np , so $E[Y | X] = X \times 0.4$.
 - Next, apply the Law of Iterated Expectation: $E[Y] = E[E[Y | X]] = E[0.4X]$.
 - Use linearity of expectation: $E[Y] = 0.4E[X]$.
 - Compute $E[X] = (1 \times 0.6) + (2 \times 0.4) = 0.6 + 0.8 = 1.4$.
 - Finally, $E[Y] = 0.4 \times 1.4 = 0.56$.

Answer: 0.56

2. Suppose X is uniformly distributed on the interval $[0, 10]$. Given $X = x$, Y is uniformly distributed on the interval $[0, x]$. Find $E[Y]$.
 - Find the conditional expectation $E[Y | X = x]$. Since Y is uniform on $[0, x]$, the mean is $x/2$.
 - Thus, $E[Y | X] = X/2$.
 - Apply the Law of Iterated Expectation: $E[Y] = E[X/2] = \frac{1}{2}E[X]$.
 - Since X is uniform on $[0, 10]$, $E[X] = (0 + 10)/2 = 5$.
 - Therefore, $E[Y] = \frac{1}{2} \times 5 = 2.5$.

Answer: 2.5

3. Let X follow a Poisson distribution with $\lambda = 3$. Given X , Y follows a Poisson distribution with parameter $\lambda = X$. Find $E[Y]$.
 - Find the conditional expectation $E[Y | X]$. Since Y given X is Poisson with parameter X , $E[Y | X] = X$.
 - Apply the Law of Iterated Expectation: $E[Y] = E[E[Y | X]] = E[X]$.
 - Since X is Poisson with $\lambda = 3$, $E[X] = 3$.
 - Therefore, $E[Y] = 3$.

Answer: 3