Expectation and variance of sums — the key identities

Lesson 61 · Module 4 — Joint Distributions & Dependence · Probability & Statistics · daily-maths.uk/probstat

Practice problems

1. Two random variables X and Y have variances $\text{Var}(X) = 5$ and $\text{Var}(Y) = 8$. Their covariance is $\text{Cov}(X, Y) = 3$. Calculate $\text{Var}(X + Y)$.

2. Suppose X and Y are independent random variables with $\text{Var}(X) = 12$ and $\text{Var}(Y) = 7$. Find $\text{Var}(X + Y)$.

3. Two variables X and Y have $\text{Var}(X) = 9$, $\text{Var}(Y) = 16$, and a correlation coefficient $\rho = -0.5$. Find $\text{Var}(X + Y)$.

Solutions

1. Two random variables X and Y have variances $\text{Var}(X) = 5$ and $\text{Var}(Y) = 8$. Their covariance is $\text{Cov}(X, Y) = 3$. Calculate $\text{Var}(X + Y)$.

   - Identify the formula for the variance of a sum: $\text{Var}(X+Y) = \text{Var}(X)+\text{Var}(Y)+2\text{Cov}(X, Y)$.
   - Substitute the given values: $5 + 8 + 2(3)$.
   - Calculate the sum: $13 + 6 = 19$.

   Answer: 19

2. Suppose X and Y are independent random variables with $\text{Var}(X) = 12$ and $\text{Var}(Y) = 7$. Find $\text{Var}(X + Y)$.

   - Since X and Y are independent, their covariance $\text{Cov}(X, Y) = 0$.
   - The formula simplifies to $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$.
   - Substitute the values: $12 + 7 = 19$.

   Answer: 19

3. Two variables X and Y have $\text{Var}(X) = 9$, $\text{Var}(Y) = 16$, and a correlation coefficient $\rho = -0.5$. Find $\text{Var}(X + Y)$.

   - First, find the covariance using $\text{Cov}(X, Y) = \rho\sigma_X\sigma_Y$.
   - Calculate standard deviations: $\sigma_X = \sqrt{9} = 3$ and $\sigma_Y = \sqrt{16} = 4$.
   - Compute covariance: $-0.5 \times 3 \times 4 = -6$.
   - Apply the variance of sums formula: $9 + 16 + 2(-6)$.
   - Calculate the final result: $25 - 12 = 13$.

   Answer: 13