

Sums of Random Variables; Convolution Intuition

Lesson 60 · Module 4 — Joint Distributions & Dependence · Probability & Statistics · daily-maths.uk/probstat

Practice problems

1. Let X and Y be independent random variables. X takes values 0 and 1 with probabilities $P(X = 0) = 0.4$ and $P(X = 1) = 0.6$. Y takes values 1 and 2 with probabilities $P(Y = 1) = 0.7$ and $P(Y = 2) = 0.3$. Find the probability mass function of $Z = X + Y$.
2. Two fair four-sided dice (numbered 1 to 4) are rolled. Let Z be the sum of the results. What is $P(Z = 5)$?
3. Suppose X and Y are dependent variables with joint PMF $P(X = 1, Y = 1) = 0.1$, $P(X = 1, Y = 2) = 0.4$, $P(X = 2, Y = 1) = 0.3$, and $P(X = 2, Y = 2) = 0.2$. Find the variance of $Z = X + Y$.

Solutions

1. Let X and Y be independent random variables. X takes values 0 and 1 with probabilities $P(X = 0) = 0.4$ and $P(X = 1) = 0.6$. Y takes values 1 and 2 with probabilities $P(Y = 1) = 0.7$ and $P(Y = 2) = 0.3$. Find the probability mass function of $Z = X + Y$.

- Identify possible values of Z : $0 + 1 = 1$, $0 + 2 = 2$, $1 + 1 = 2$, $1 + 2 = 3$. So $Z \in \{1, 2, 3\}$.
- Compute $P(Z = 1)$: Only $X = 0$ and $Y = 1$ works. $0.4 \times 0.7 = 0.28$.
- Compute $P(Z = 2)$: $X = 0, Y = 2$ or $X = 1, Y = 1$. $(0.4 \times 0.3) + (0.6 \times 0.7) = 0.12 + 0.42 = 0.54$.
- Compute $P(Z = 3)$: Only $X = 1$ and $Y = 2$ works. $0.6 \times 0.3 = 0.18$.
- Check sum: $0.28 + 0.54 + 0.18 = 1.0$.

Answer: $P(Z=1)=0.28$, $P(Z=2)=0.54$, $P(Z=3)=0.18$

2. Two fair four-sided dice (numbered 1 to 4) are rolled. Let Z be the sum of the results. What is $P(Z = 5)$?

- The total number of outcomes is $4 \times 4 = 16$.
- List pairs (x, y) that sum to 5: $(1, 4), (2, 3), (3, 2), (4, 1)$.
- There are 4 such pairs.
- The probability is $4/16 = 1/4$.

Answer: 0.25

3. Suppose X and Y are dependent variables with joint PMF $P(X = 1, Y = 1) = 0.1$, $P(X = 1, Y = 2) = 0.4$, $P(X = 2, Y = 1) = 0.3$, and $P(X = 2, Y = 2) = 0.2$. Find the variance of $Z = X + Y$.

- Find PMF of Z : $P(Z = 2) = 0.1$, $P(Z = 3) = 0.4 + 0.3 = 0.7$, $P(Z = 4) = 0.2$.
- Compute $E[Z] = (2 \times 0.1) + (3 \times 0.7) + (4 \times 0.2) = 0.2 + 2.1 + 0.8 = 3.1$.
- Compute $E[Z^2] = (2^2 \times 0.1) + (3^2 \times 0.7) + (4^2 \times 0.2) = 0.4 + 6.3 + 3.2 = 9.9$.
- Compute $\text{Var}(Z) = E[Z^2] - (E[Z])^2 = 9.9 - (3.1)^2 = 9.9 - 9.61 = 0.29$.

Answer: 0.29