

Correlation — the standardised story (and its limits)

Lesson 59 · Module 4 — Joint Distributions & Dependence · Probability & Statistics · daily-maths.uk/probstat

Practice problems

1. Given $\text{Cov}(X, Y) = 12$, $\sigma_X = 4$, and $\sigma_Y = 3$, calculate the correlation coefficient $\rho_{X,Y}$.
2. Two variables X and Y have a correlation of $\rho = -0.7$. If the standard deviation of X is doubled while all other properties remain the same, what happens to the correlation coefficient?
3. Let X be a random variable with mean 0 and variance 1. Let $Y = X^2$. Show that $\rho_{X,Y} = 0$ if the distribution of X is symmetric about 0.

Solutions

1. Given $\text{Cov}(X, Y) = 12$, $\sigma_X = 4$, and $\sigma_Y = 3$, calculate the correlation coefficient $\rho_{X,Y}$.

- Use the formula $\rho = \frac{\text{Cov}(X,Y)}{\sigma_X \sigma_Y}$.
- Substitute the values: $\rho = \frac{12}{4 \times 3}$.
- Calculate the denominator: $4 \times 3 = 12$.
- Divide: $12/12 = 1$.

Answer: 1

2. Two variables X and Y have a correlation of $\rho = -0.7$. If the standard deviation of X is doubled while all other properties remain the same, what happens to the correlation coefficient?

- The correlation formula is $\rho = \frac{\text{Cov}(X,Y)}{\sigma_X \sigma_Y}$.
- If σ_X is doubled, the covariance $\text{Cov}(X, Y) = E[(X - \mu_X)(Y - \mu_Y)]$ also doubles because the deviations $(X - \mu_X)$ are doubled.
- The new correlation is $\rho_{\text{new}} = \frac{2 \times \text{Cov}(X,Y)}{(2\sigma_X)\sigma_Y}$.
- The factor of 2 in the numerator and denominator cancels out.

Answer: The correlation remains -0.7.

3. Let X be a random variable with mean 0 and variance 1. Let $Y = X^2$. Show that $\rho_{X,Y} = 0$ if the distribution of X is symmetric about 0.

- The correlation is $\rho = \frac{\text{Cov}(X,Y)}{\sigma_X \sigma_Y}$.
- $\text{Cov}(X, Y) = E[XY] - E[X]E[Y]$.
- Since $E[X] = 0$, $\text{Cov}(X, Y) = E[X \cdot X^2] = E[X^3]$.
- For a symmetric distribution about 0, the expected value of any odd power of X is 0.
- Therefore, $\text{Cov}(X, Y) = 0$, which implies $\rho = 0$.

Answer: $\rho = 0$