

Covariance — do they move together?

Lesson 58 · Module 4 — Joint Distributions & Dependence · Probability & Statistics · daily-maths.uk/probstat

Practice problems

1. Given the joint probabilities $p(0,0) = 0.2, p(0,1) = 0.3, p(1,0) = 0.1, p(1,1) = 0.4$, calculate $\text{Cov}(X, Y)$.
2. If X and Y are independent random variables with $\text{Var}(X) = 2$ and $\text{Var}(Y) = 3$, what is $\text{Cov}(X, Y)$?
3. Suppose X and Y have a joint PDF $f(x, y) = 4xy$ for $0 \leq x, y \leq 1$ and 0 otherwise. Calculate $\text{Cov}(X, Y)$.

Solutions

1. Given the joint probabilities $p(0,0) = 0.2, p(0,1) = 0.3, p(1,0) = 0.1, p(1,1) = 0.4$, calculate $\text{Cov}(X, Y)$.

- Find marginals: $p_X(0) = 0.5, p_X(1) = 0.5$ and $p_Y(0) = 0.3, p_Y(1) = 0.7$.
- Compute means: $E[X] = 0.5$ and $E[Y] = 0.7$.
- Compute $E[XY]$: only $p(1,1)$ contributes, so $E[XY] = 1 \cdot 1 \cdot 0.4 = 0.4$.
- Apply formula: $\text{Cov}(X, Y) = 0.4 - (0.5)(0.7) = 0.4 - 0.35 = 0.05$.

Answer: 0.05

2. If X and Y are independent random variables with $\text{Var}(X) = 2$ and $\text{Var}(Y) = 3$, what is $\text{Cov}(X, Y)$?

- Recall that for independent variables, $E[XY] = E[X]E[Y]$.
- Substitute this into the covariance formula: $\text{Cov}(X, Y) = E[XY] - E[X]E[Y] = 0$.

Answer: 0

3. Suppose X and Y have a joint PDF $f(x, y) = 4xy$ for $0 \leq x, y \leq 1$ and 0 otherwise. Calculate $\text{Cov}(X, Y)$.

- Find marginal $f_X(x) = \int_0^1 4xy \, dy = [2xy^2]_0^1 = 2x$.
- Compute $E[X] = \int_0^1 x(2x) \, dx = [2x^3/3]_0^1 = 2/3$. By symmetry, $E[Y] = 2/3$.
- Compute $E[XY] = \int_0^1 \int_0^1 xy(4xy) \, dx \, dy = \int_0^1 \int_0^1 4x^2y^2 \, dx \, dy = \int_0^1 [4x^3y^2/3]_0^1 \, dy = \int_0^1 4y^2/3 \, dy = [4y^3/9]_0^1 = 4/9$.
- Apply formula: $\text{Cov}(X, Y) = 4/9 - (2/3)(2/3) = 4/9 - 4/9 = 0$.

Answer: 0