

Independence of Random Variables

Lesson 57 · Module 4 — Joint Distributions & Dependence · Probability & Statistics · daily-maths.uk/probstat

Practice problems

1. Given the joint PMF $p(0,0) = 0.2, p(0,1) = 0.2, p(1,0) = 0.1, p(1,1) = 0.5$, are X and Y independent?
2. A joint PDF is given by $f(x,y) = 6x^2y$ for $0 < x < 1$ and $0 < y < 1$. Are X and Y independent?
3. Consider the joint PDF $f(x,y) = 2$ for $0 < x < 1$ and $x < y < 1$. Without calculating marginals, are X and Y independent? Explain.

Solutions

1. Given the joint PMF $p(0,0) = 0.2, p(0,1) = 0.2, p(1,0) = 0.1, p(1,1) = 0.5$, are X and Y independent?

- Compute marginal $p_X(0) = 0.2 + 0.2 = 0.4$ and $p_X(1) = 0.1 + 0.5 = 0.6$.
- Compute marginal $p_Y(0) = 0.2 + 0.1 = 0.3$ and $p_Y(1) = 0.2 + 0.5 = 0.7$.
- Check $p(0,0) = 0.2$. The product $p_X(0)p_Y(0) = 0.4 \times 0.3 = 0.12$.
- Since $0.2 \neq 0.12$, the variables are dependent.

Answer: Dependent

2. A joint PDF is given by $f(x,y) = 6x^2y$ for $0 < x < 1$ and $0 < y < 1$. Are X and Y independent?

- Find marginal $f_X(x) = \int_0^1 6x^2y \, dy = [3x^2y^2]_0^1 = 3x^2$.
- Find marginal $f_Y(y) = \int_0^1 6x^2y \, dx = [2x^3y]_0^1 = 2y$.
- Check product: $f_X(x)f_Y(y) = 3x^2 \times 2y = 6x^2y$.
- The product equals the joint PDF and the support is a rectangle, so they are independent.

Answer: Independent

3. Consider the joint PDF $f(x,y) = 2$ for $0 < x < 1$ and $x < y < 1$. Without calculating marginals, are X and Y independent? Explain.

- Identify the support of the distribution: $S = \{(x,y) : 0 < x < 1, x < y < 1\}$.
- Observe that the range of y depends on the value of x (specifically, y must be greater than x).
- Because the support is a triangle and not a rectangle aligned with the axes, the variables are dependent.

Answer: Dependent