Marginal and Conditional Distributions

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Practice problems

1. Given the joint probability table $p(x, y)$ where $x \in \{0, 1, 2\}$ and $y \in \{0, 1, 2\}$ with values $p(0,0) = 0.1, p(0,1) = 0.1, p(0,2) = 0.1, p(1,0) = 0.1, p(1,1) = 0.2, p(1,2) = 0.1, p(2,0) = 0.1, p(2,1) = 0.1, p(2,2) = 0.1$, find the marginal distribution $p_X(x)$ and the conditional probability $p(X = 1|Y = 1)$.

2. A joint PDF is given by $f(x, y) = 4xy$ for $0 < x < 1$ and $0 < y < 1$. Find the marginal PDF $f_X(x)$ and the conditional PDF $f_{Y|X}(y|x)$.

3. If X and Y are independent random variables with marginal PDFs $f_X(x) = e^{-x}$ for $x > 0$ and $f_Y(y) = 2e^{-2y}$ for $y > 0$, what is the conditional PDF $f_{X|Y}(x|y)$?

Solutions

1. Given the joint probability table $p(x, y)$ where $x \in \{0, 1, 2\}$ and $y \in \{0, 1, 2\}$ with values $p(0, 0) = 0.1, p(0, 1) = 0.1, p(0, 2) = 0.1, p(1, 0) = 0.1, p(1, 1) = 0.2, p(1, 2) = 0.1, p(2, 0) = 0.1, p(2, 1) = 0.1, p(2, 2) = 0.1$, find the marginal distribution $p_X(x)$ and the conditional probability $p(X = 1|Y = 1)$.

   - Sum the rows to find $p_X(x)$: $p_X(0) = 0.1 + 0.1 + 0.1 = 0.3$, $p_X(1) = 0.1 + 0.2 + 0.1 = 0.4$, $p_X(2) = 0.1 + 0.1 + 0.1 = 0.3$.
   - Check sum: $0.3 + 0.4 + 0.3 = 1.0$.
   - For $p(X = 1|Y = 1)$, find the marginal $p_Y(1) = p(0, 1) + p(1, 1) + p(2, 1) = 0.1 + 0.2 + 0.1 = 0.4$.
   - Then $p(X = 1|Y = 1) = p(1, 1)/p_Y(1) = 0.2/0.4 = 0.5$.

   Answer: p_X(0)=0.3, p_X(1)=0.4, p_X(2)=0.3; p(X=1—Y=1)=0.5

2. A joint PDF is given by $f(x, y) = 4xy$ for $0 < x < 1$ and $0 < y < 1$. Find the marginal PDF $f_X(x)$ and the conditional PDF $f_{Y|X}(y|x)$.

   - Integrate $f(x, y)$ over y: $f_X(x) = \int_0^1 4xy\, dy = [2xy^2]_0^1 = 2x$.
   - Check: $\int_0^1 2x\, dx = [x^2]_0^1 = 1$. Correct.
   - Find conditional: $f_{Y|X}(y|x) = f(x, y)/f_X(x) = (4xy)/(2x) = 2y$.

   Answer: f_X(x)=2x, f_Y—X(y—x)=2y

3. If X and Y are independent random variables with marginal PDFs $f_X(x) = e^{-x}$ for $x > 0$ and $f_Y(y) = 2e^{-2y}$ for $y > 0$, what is the conditional PDF $f_{X|Y}(x|y)$?

   - Since X and Y are independent, the conditional distribution of X given Y is simply the marginal distribution of X.
   - Therefore, $f_{X|Y}(x|y) = f_X(x) = e^{-x}$ for $x > 0$.

   Answer: f_X—Y(x—y)=eˆ-x