

Joint distributions — randomness in two dimensions

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Practice problems

1. Given the joint PMF $p(x, y) = \frac{x+y}{21}$ for $x \in \{1, 2\}$ and $y \in \{1, 2, 3\}$, find the marginal PMF $p_X(x)$ for $x = 1$ and $x = 2$.
2. A joint PDF is given by $f(x, y) = 2$ for $0 \leq x \leq 1$ and $0 \leq y \leq x$, and 0 otherwise. Find the marginal PDF $f_Y(y)$.
3. For the joint PDF $f(x, y) = 8xy$ for $0 \leq y \leq x \leq 1$, calculate the probability $P(X < 0.5)$.

Solutions

1. Given the joint PMF $p(x, y) = \frac{x+y}{21}$ for $x \in \{1, 2\}$ and $y \in \{1, 2, 3\}$, find the marginal PMF $p_X(x)$ for $x = 1$ and $x = 2$.

- For $x = 1$, sum $p(1, y)$ for $y = 1, 2, 3$: $\frac{1+1}{21} + \frac{1+2}{21} + \frac{1+3}{21} = \frac{2+3+4}{21} = \frac{9}{21} = 3/7$.
- For $x = 2$, sum $p(2, y)$ for $y = 1, 2, 3$: $\frac{2+1}{21} + \frac{2+2}{21} + \frac{2+3}{21} = \frac{3+4+5}{21} = \frac{12}{21} = 4/7$.

Answer: $p_X(1) = 3/7, p_X(2) = 4/7$

2. A joint PDF is given by $f(x, y) = 2$ for $0 \leq x \leq 1$ and $0 \leq y \leq x$, and 0 otherwise. Find the marginal PDF $f_Y(y)$.

- The region is a triangle with vertices $(0, 0), (1, 0), (1, 1)$.
- To find $f_Y(y)$, we integrate $f(x, y)$ with respect to x .
- For a fixed y , x ranges from y to 1.
- $\int_y^1 2 dx = [2x]_y^1 = 2(1 - y)$.
- The marginal PDF is $f_Y(y) = 2(1 - y)$ for $0 \leq y \leq 1$.

Answer: $f_Y(y) = 2(1 - y)$ for $0 \leq y \leq 1$

3. For the joint PDF $f(x, y) = 8xy$ for $0 \leq y \leq x \leq 1$, calculate the probability $P(X < 0.5)$.

- We integrate the joint PDF over the region where $0 \leq x \leq 0.5$ and $0 \leq y \leq x$.
- Inner integral: $\int_0^x 8xy dy = [4xy^2]_0^x = 4x^3$.
- Outer integral: $\int_0^{0.5} 4x^3 dx = [x^4]_0^{0.5} = (0.5)^4 = 0.0625$.

Answer: 0.0625