

Module 3 Review + Self-Test

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Practice problems

1. Find the value of the constant k such that $f(x) = kx^3$ for $1 \leq x \leq 2$ (and 0 otherwise) is a valid probability density function.
2. For an exponential distribution with rate $\lambda = 0.5$, find the median waiting time.
3. Let X be a uniform random variable on $[0, 1]$. Find the PDF of $Y = X^2$.

Solutions

1. Find the value of the constant k such that $f(x) = kx^3$ for $1 \leq x \leq 2$ (and 0 otherwise) is a valid probability density function.

- Set the total integral equal to 1: $\int_1^2 kx^3 dx = 1$.
- Compute the integral: $k[\frac{1}{4}x^4]_1^2 = 1$.
- Evaluate the bounds: $k(\frac{16}{4} - \frac{1}{4}) = 1$.
- Solve for k : $k(\frac{15}{4}) = 1 \implies k = \frac{4}{15}$.

Answer: $k = 4/15$

2. For an exponential distribution with rate $\lambda = 0.5$, find the median waiting time.

- The CDF of an exponential distribution is $F(x) = 1 - e^{-\lambda x}$.
- Set $F(m) = 0.5$: $1 - e^{-0.5m} = 0.5$.
- Rearrange: $e^{-0.5m} = 0.5$.
- Take the natural log: $-0.5m = \ln(0.5)$.
- Solve for m : $m = \frac{\ln(0.5)}{-0.5} = -2\ln(0.5) = 2\ln(2) \approx 1.386$.

Answer: $m \approx 1.386$

3. Let X be a uniform random variable on $[0, 1]$. Find the PDF of $Y = X^2$.

- Find the CDF of Y : $F_Y(y) = P(X^2 \leq y) = P(X \leq \sqrt{y})$.
- Since X is uniform on $[0, 1]$, $P(X \leq \sqrt{y}) = \sqrt{y}$ for $0 \leq y \leq 1$.
- Differentiate the CDF to find the PDF: $f_Y(y) = \frac{d}{dy}\sqrt{y}$.
- The result is $f_Y(y) = \frac{1}{2\sqrt{y}}$ for $0 < y \leq 1$.

Answer: $f_Y(y) = \frac{1}{2\sqrt{y}}$