

Quantiles, percentiles, and the median

Lesson 53 · Module 3 — Continuous Random Variables · Probability & Statistics · daily-maths.uk/probstat

Practice problems

1. Find the 75th percentile of an exponential distribution with rate $\lambda = 2$.
2. A random variable X is uniformly distributed on the interval $[20, 100]$. Find the median.
3. For an exponential distribution with rate $\lambda = 1$, find the value x such that $P(X > x) = 0.10$.

Solutions

1. Find the 75th percentile of an exponential distribution with rate $\lambda = 2$.

- Set the CDF equal to 0.75: $1 - e^{-2x} = 0.75$.
- Rearrange to isolate the exponential: $e^{-2x} = 0.25$.
- Take the natural logarithm of both sides: $-2x = \ln(0.25)$.
- Solve for x : $x = \frac{\ln(0.25)}{-2} = \frac{-1.3863}{-2} \approx 0.6931$.

Answer: 0.6931

2. A random variable X is uniformly distributed on the interval $[20, 100]$. Find the median.

- The CDF for a uniform distribution is $F(x) = \frac{x-a}{b-a}$.
- Substitute $a = 20$ and $b = 100$: $F(x) = \frac{x-20}{80}$.
- Set $F(x) = 0.5$ for the median: $\frac{x-20}{80} = 0.5$.
- Multiply by 80: $x - 20 = 40$.
- Solve for x : $x = 60$.

Answer: 60

3. For an exponential distribution with rate $\lambda = 1$, find the value x such that $P(X > x) = 0.10$.

- The condition $P(X > x) = 0.10$ is equivalent to $P(X \leq x) = 0.90$.
- Set the CDF equal to 0.90: $1 - e^{-x} = 0.90$.
- Isolate the exponential: $e^{-x} = 0.10$.
- Take the natural logarithm: $-x = \ln(0.10)$.
- Solve for x : $x = -(-2.3026) = 2.3026$.

Answer: 2.3026