

Transformations of Random Variables

Lesson 51 · Module 3 — Continuous Random Variables · Probability & Statistics · daily-maths.uk/probstat

Practice problems

1. Let X be a random variable uniformly distributed on $[0, 1]$. Find the PDF of $Y = X^3$.
2. Let X be uniformly distributed on $[0, 2]$. Find the PDF of $Y = e^X$.
3. Let X be uniformly distributed on $[1, 2]$. Find the PDF of $Y = 1/X$.

Solutions

1. Let X be a random variable uniformly distributed on $[0, 1]$. Find the PDF of $Y = X^3$.

- The PDF of X is $f_X(x) = 1$ for $0 \leq x \leq 1$.
- The range of Y is $[0^3, 1^3] = [0, 1]$.
- The CDF of Y is $F_Y(y) = P(X^3 \leq y) = P(X \leq y^{1/3}) = y^{1/3}$.
- The PDF is the derivative: $f_Y(y) = \frac{d}{dy} y^{1/3} = \frac{1}{3} y^{-2/3}$.

Answer: $f_Y(y) = \frac{1}{3} y^{-2/3}$ for $0 \leq y \leq 1$

2. Let X be uniformly distributed on $[0, 2]$. Find the PDF of $Y = e^X$.

- The PDF of X is $f_X(x) = 1/2$ for $0 \leq x \leq 2$.
- The range of Y is $[e^0, e^2] = [1, e^2]$.
- The CDF of Y is $F_Y(y) = P(e^X \leq y) = P(X \leq \ln y)$.
- Since X is uniform on $[0, 2]$, $F_X(x) = x/2$. Thus $F_Y(y) = \frac{\ln y}{2}$.
- The PDF is $f_Y(y) = \frac{d}{dy} \frac{\ln y}{2} = \frac{1}{2y}$.

Answer: $f_Y(y) = \frac{1}{2y}$ for $1 \leq y \leq e^2$

3. Let X be uniformly distributed on $[1, 2]$. Find the PDF of $Y = 1/X$.

- The PDF of X is $f_X(x) = 1$ for $1 \leq x \leq 2$.
- The range of Y is $[1/2, 1/1] = [1/2, 1]$.
- The CDF of Y is $F_Y(y) = P(1/X \leq y) = P(X \geq 1/y)$.
- This is $1 - P(X < 1/y) = 1 - F_X(1/y)$.
- Since $F_X(x) = x - 1$ for $x \in [1, 2]$, we have $F_Y(y) = 1 - (1/y - 1) = 2 - 1/y$.
- The PDF is $f_Y(y) = \frac{d}{dy} (2 - 1/y) = 1/y^2$.

Answer: $f_Y(y) = \frac{1}{y^2}$ for $1/2 \leq y \leq 1$