

Standardisation and the z-table; 68-95-99.7

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Practice problems

1. A random variable X follows a normal distribution with mean $\mu = 50$ and standard deviation $\sigma = 5$. Find the probability $P(X > 62)$.
2. For a normal distribution with $\mu = 100$ and $\sigma = 15$, use the empirical rule to estimate the probability that a value falls between 70 and 130.
3. A population is normally distributed with $\mu = 200$ and $\sigma = 20$. Find the value x such that $P(X < x) = 0.025$.

Solutions

1. A random variable X follows a normal distribution with mean $\mu = 50$ and standard deviation $\sigma = 5$. Find the probability $P(X > 62)$.

- Calculate the z-score: $z = \frac{62-50}{5} = \frac{12}{5} = 2.4$.
- Find the cumulative probability from the z-table: $P(Z < 2.4) \approx 0.9918$.
- Since we want the probability of being greater than 62, subtract from 1: $P(X > 62) = 1 - 0.9918 = 0.0082$.

Answer: 0.0082

2. For a normal distribution with $\mu = 100$ and $\sigma = 15$, use the empirical rule to estimate the probability that a value falls between 70 and 130.

- Determine how many standard deviations 70 and 130 are from the mean.
- For 70: $z = \frac{70-100}{15} = -2$.
- For 130: $z = \frac{130-100}{15} = 2$.
- The empirical rule states that approximately 95.45% of data falls within 2 standard deviations.

Answer: 0.9545

3. A population is normally distributed with $\mu = 200$ and $\sigma = 20$. Find the value x such that $P(X < x) = 0.025$.

- Find the z-score that corresponds to a cumulative area of 0.025 in the z-table.
- The z-score for 0.025 is $z = -1.96$.
- Use the formula $x = \mu + z\sigma$.
- Calculate $x = 200 + (-1.96)(20) = 200 - 39.2 = 160.8$.

Answer: 160.8