The normal distribution I — meet the bell curve

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Practice problems

1. A random variable X follows a normal distribution with mean $\mu = 50$ and standard deviation $\sigma = 4$. Find the interval that contains the middle 95% of the probability.

2. The weights of apples in an orchard are normally distributed with $\mu = 150$ grams and $\sigma = 12$ grams. What percentage of apples weigh more than 174 grams?

3. A process produces parts with a normal distribution of diameters. The variance is 0.0016 mm^2 and the mean is 10.00 mm. If a part is rejected for being more than 3 standard deviations from the mean, what is the range of acceptable diameters?

Solutions

1. A random variable X follows a normal distribution with mean $\mu = 50$ and standard deviation $\sigma = 4$. Find the interval that contains the middle 95% of the probability.

   - The empirical rule states that the middle 95% of a normal distribution lies within 2 standard deviations of the mean.
   - Calculate the lower bound: $50 - 2(4) = 50 - 8 = 42$.
   - Calculate the upper bound: $50 + 2(4) = 50 + 8 = 58$.

   Answer: [42, 58]

2. The weights of apples in an orchard are normally distributed with $\mu = 150$ grams and $\sigma = 12$ grams. What percentage of apples weigh more than 174 grams?

   - First, find the z-score for 174 grams: $z = (174 - 150)/12 = 24/12 = 2$.
   - The value 174 is exactly 2 standard deviations above the mean.
   - The empirical rule says 95% of data is within 2 standard deviations, meaning 5% is outside this range.
   - Because the curve is symmetric, half of that 5% is in the upper tail: $5\%/2 = 2.5\%$.

   Answer: 2.5%

3. A process produces parts with a normal distribution of diameters. The variance is 0.0016 mm^2 and the mean is 10.00 mm. If a part is rejected for being more than 3 standard deviations from the mean, what is the range of acceptable diameters?

   - First, find the standard deviation σ from the variance: $\sigma = \sqrt{0.0016} = 0.04$ mm.
   - The acceptable range is $\mu \pm 3\sigma$.
   - Lower bound: $10.00 - 3(0.04) = 10.00 - 0.12 = 9.88$ mm.
   - Upper bound: $10.00 + 3(0.04) = 10.00 + 0.12 = 10.12$ mm.

   Answer: [9.88, 10.12]