

Exponential and Poisson: two views of one process

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Practice problems

1. A computer server crashes an average of 2 times per month. What is the probability that the server goes at least 3 months without a crash?
2. Customers arrive at a bank at a rate of 12 per hour. What is the probability that exactly 4 customers arrive in a 15-minute interval?
3. A lightbulb fails at a rate of 0.1 times per year. What is the probability that the second failure occurs after 20 years?

Solutions

1. A computer server crashes an average of 2 times per month. What is the probability that the server goes at least 3 months without a crash?

- Identify the rate $\lambda = 2$ crashes per month.
- We want the probability that the time until the first crash T is greater than 3.
- Use the exponential survival function $P(T > t) = e^{-\lambda t}$.
- Substitute $\lambda = 2$ and $t = 3$ to get $e^{-(2)(3)} = e^{-6}$.

Answer: $e^{-6} \approx 0.0025$

2. Customers arrive at a bank at a rate of 12 per hour. What is the probability that exactly 4 customers arrive in a 15-minute interval?

- Convert the rate to the given time unit: $\lambda = 12$ per hour is 3 per 15 minutes.
- Alternatively, $\mu = \lambda t = 12 \times (15/60) = 3$.
- Use the Poisson PMF for $k = 4$ and $\mu = 3$.
- Calculate $P(X = 4) = \frac{e^{-3}3^4}{4!} = \frac{81e^{-3}}{24} = \frac{27}{8}e^{-3}$.

Answer: $\frac{27}{8}e^{-3} \approx 0.1680$

3. A lightbulb fails at a rate of 0.1 times per year. What is the probability that the second failure occurs after 20 years?

- The second failure occurs after 20 years if and only if there are 0 or 1 failures in the first 20 years.
- Calculate the expected number of failures: $\mu = \lambda t = 0.1 \times 20 = 2$.
- Use the Poisson distribution to find $P(X = 0) + P(X = 1)$.
- Compute $e^{-2}\frac{2^0}{0!} + e^{-2}\frac{2^1}{1!} = e^{-2} + 2e^{-2} = 3e^{-2}$.

Answer: $3e^{-2} \approx 0.4060$