

The Exponential Distribution — Memorylessness

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Practice problems

1. A lightbulb has a mean lifetime of 100 hours. Assuming the lifetime follows an exponential distribution, what is the probability that the bulb lasts more than 150 hours?
2. The time between arrivals of emails to a server follows an exponential distribution with a rate of $\lambda = 2$ emails per minute. What is the probability that the next email arrives within the first 30 seconds?
3. A radioactive isotope has a mean decay time of 500 years. Given that a specific atom has already survived for 200 years, what is the probability it will survive for at least another 500 years?

Solutions

1. A lightbulb has a mean lifetime of 100 hours. Assuming the lifetime follows an exponential distribution, what is the probability that the bulb lasts more than 150 hours?

- Identify the mean $E[X] = 100$.
- Calculate the rate $\lambda = 1/100 = 0.01$.
- Use the survival function $P(X > x) = e^{-\lambda x}$.
- Substitute $x = 150$ and $\lambda = 0.01$: $P(X > 150) = e^{-0.01 \cdot 150} = e^{-1.5}$.

Answer: $e^{-1.5} \approx 0.2231$

2. The time between arrivals of emails to a server follows an exponential distribution with a rate of $\lambda = 2$ emails per minute. What is the probability that the next email arrives within the first 30 seconds?

- Convert the time to minutes to match the rate: 30 seconds = 0.5 minutes.
- Use the CDF $F(x) = 1 - e^{-\lambda x}$.
- Substitute $\lambda = 2$ and $x = 0.5$: $F(0.5) = 1 - e^{-2 \cdot 0.5} = 1 - e^{-1}$.

Answer: $1 - e^{-1} \approx 0.6321$

3. A radioactive isotope has a mean decay time of 500 years. Given that a specific atom has already survived for 200 years, what is the probability it will survive for at least another 500 years?

- Identify the mean $E[X] = 500$, so $\lambda = 1/500$.
- Recognize the memoryless property: $P(X > 200 + 500 \mid X > 200) = P(X > 500)$.
- Calculate $P(X > 500) = e^{-\lambda \cdot 500} = e^{-(1/500) \cdot 500} = e^{-1}$.

Answer: $e^{-1} \approx 0.3679$