

The uniform distribution

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Practice problems

1. A random variable X is uniformly distributed on the interval $[5, 15]$. Find the probability $P(8 \leq X \leq 12)$.
2. Find the mean and variance of a uniform random variable X distributed on the interval $[10, 20]$.
3. A uniform random variable X on the interval $[a, b]$ has a mean of 10 and a variance of 3. Find the values of a and b .

Solutions

1. A random variable X is uniformly distributed on the interval $[5, 15]$. Find the probability $P(8 \leq X \leq 12)$.

- Identify the parameters: $a = 5$ and $b = 15$.
- Calculate the total width: $b - a = 15 - 5 = 10$.
- Calculate the target width: $12 - 8 = 4$.
- The probability is the ratio of the target width to the total width: $4/10 = 0.4$.

Answer: 0.4

2. Find the mean and variance of a uniform random variable X distributed on the interval $[10, 20]$.

- The mean is the average of the endpoints: $E[X] = (10 + 20)/2 = 30/2 = 15$.
- The variance is the square of the width divided by 12: $Var(X) = (20 - 10)^2/12$.
- Compute the square: $10^2 = 100$.
- Divide by 12: $100/12 = 25/3 \approx 8.33$.

Answer: $E[X] = 15$, $Var(X) = 8.33$

3. A uniform random variable X on the interval $[a, b]$ has a mean of 10 and a variance of 3. Find the values of a and b .

- From the mean: $(a + b)/2 = 10 \implies a + b = 20$.
- From the variance: $(b - a)^2/12 = 3 \implies (b - a)^2 = 36$.
- Taking the square root: $b - a = 6$.
- Solve the system: $a + b = 20$ and $b - a = 6$. Adding the equations gives $2b = 26$, so $b = 13$.
- Subtracting gives $2a = 14$, so $a = 7$.

Answer: $a = 7$, $b = 13$