

Expectation and variance of continuous variables

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Practice problems

1. A continuous random variable X has the PDF $f(x) = \frac{3}{2}x^2$ for $0 \leq x \leq 1$ and 0 otherwise. Find the mean $E[X]$ and the variance $Var(X)$.
2. A random variable X has a PDF $f(x) = 2 - 2x$ for $0 \leq x \leq 1$. Calculate the expectation $E[X]$.
3. For a random variable X with PDF $f(x) = \frac{1}{2}e^{-x/2}$ for $x \geq 0$, find the variance $Var(X)$.

Solutions

1. A continuous random variable X has the PDF $f(x) = \frac{3}{2}x^2$ for $0 \leq x \leq 1$ and 0 otherwise. Find the mean $E[X]$ and the variance $Var(X)$.

- Compute the mean: $E[X] = \int_0^1 x(\frac{3}{2}x^2) dx = \int_0^1 \frac{3}{2}x^3 dx = [\frac{3}{8}x^4]_0^1 = \frac{3}{8}$.
- Compute the second moment: $E[X^2] = \int_0^1 x^2(\frac{3}{2}x^2) dx = \int_0^1 \frac{3}{2}x^4 dx = [\frac{3}{10}x^5]_0^1 = \frac{3}{10}$.
- Compute variance: $Var(X) = E[X^2] - (E[X])^2 = \frac{3}{10} - (\frac{3}{8})^2 = \frac{3}{10} - \frac{9}{64}$.
- Find common denominator: $\frac{96}{320} - \frac{45}{320} = \frac{51}{320}$.

Answer: $E[X] = \frac{3}{8}$, $Var(X) = \frac{51}{320}$

2. A random variable X has a PDF $f(x) = 2 - 2x$ for $0 \leq x \leq 1$. Calculate the expectation $E[X]$.

- Set up the integral: $E[X] = \int_0^1 x(2 - 2x) dx$.
- Expand the integrand: $\int_0^1 (2x - 2x^2) dx$.
- Integrate: $[x^2 - \frac{2}{3}x^3]_0^1$.
- Evaluate: $1 - \frac{2}{3} = \frac{1}{3}$.

Answer: $E[X] = \frac{1}{3}$

3. For a random variable X with PDF $f(x) = \frac{1}{2}e^{-x/2}$ for $x \geq 0$, find the variance $Var(X)$.

- Find $E[X]$ using integration by parts: $\int_0^\infty x\frac{1}{2}e^{-x/2} dx = 2$.
- Find $E[X^2]$ using integration by parts twice: $\int_0^\infty x^2\frac{1}{2}e^{-x/2} dx = 8$.
- Apply variance formula: $Var(X) = E[X^2] - (E[X])^2 = 8 - 2^2 = 8 - 4 = 4$.

Answer: $Var(X) = 4$