The pdf and CDF; probabilities as integrals

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Practice problems

1. Given the PDF $f(x) = 3x^2$ for $0 \leq x \leq 1$ and 0 otherwise, find the CDF $F(x)$.
2. A random variable has the CDF $F(x) = 1 - e^{-x}$ for $x \geq 0$. Find the PDF $f(x)$ and calculate $P(1 \leq X \leq 2)$.
3. A PDF is given by $f(x) = \frac{1}{2}x$ for $0 \leq x \leq 2$. Find the value of m such that $P(X \leq m) = 0.5$ (the median).

Solutions

1. Given the PDF $f(x) = 3x^2$ for $0 \le x \le 1$ and 0 otherwise, find the CDF $F(x)$.

   - For $x < 0$, $F(x) = 0$.
   - For $0 \le x \le 1$, $F(x) = \int_0^x 3t^2\,dt$.
   - The antiderivative of $3t^2$ is t^3.
   - Evaluating from 0 to x gives $x^3 - 0^3 = x^3$.
   - For $x > 1$, $F(x) = 1$.

   Answer: F(x) = xˆ3 \text for 0 \le x \le 1

2. A random variable has the CDF $F(x) = 1 - e^{-x}$ for $x \ge 0$. Find the PDF $f(x)$ and calculate $P(1 \le X \le 2)$.

   - The PDF is the derivative of the CDF: $f(x) = \frac{d}{dx}(1 - e^{-x}) = e^{-x}$.
   - The probability $P(1 \le X \le 2) = F(2) - F(1)$.
   - Substitute values: $(1 - e^{-2}) - (1 - e^{-1})$.
   - Simplify: $e^{-1} - e^{-2} \approx 0.3679 - 0.1353 = 0.2326$.

   Answer: f(x) = eˆ-x, P(1 \le X \le 2) \approx 0.2326

3. A PDF is given by $f(x) = \frac{1}{2}x$ for $0 \le x \le 2$. Find the value of m such that $P(X \le m) = 0.5$ (the median).

   - First, find the CDF $F(x) = \int_0^x \frac{1}{2}t\,dt = [\frac{1}{4}t^2]_0^x = \frac{1}{4}x^2$.
   - We want to find m such that $F(m) = 0.5$.
   - Set up the equation: $\frac{1}{4}m^2 = 0.5$.
   - Multiply by 4: $m^2 = 2$.
   - Solve for m: $m = \sqrt{2} \approx 1.414$.

   Answer: m = \sqrt2 \approx 1.414
