

Expected value clinic: the coupon collector

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Practice problems

1. You are collecting a set of $n = 3$ unique stickers. What is the expected number of draws to collect all three?
2. A game has $n = 5$ different character cards. If you already own 3 distinct characters, what is the expected number of additional cards you need to buy to complete the set?
3. For a set of n items, the expected time is $E[T] = nH_n$. If $n = 100$, use the approximation $H_n \approx \ln(n) + 0.577$ to estimate the expected number of draws.

Solutions

1. You are collecting a set of $n = 3$ unique stickers. What is the expected number of draws to collect all three?

- Identify $n = 3$.
- Calculate the 3rd harmonic number: $H_3 = 1 + 1/2 + 1/3 = 11/6$.
- Multiply by n : $E[T] = 3 \times (11/6) = 11/2 = 5.5$.

Answer: 5.5

2. A game has $n = 5$ different character cards. If you already own 3 distinct characters, what is the expected number of additional cards you need to buy to complete the set?

- You need the 4th and 5th characters.
- The probability of getting the 4th is $p_4 = (5-3)/5 = 2/5$. Expected time $E[T_4] = 5/2 = 2.5$.
- The probability of getting the 5th is $p_5 = (5-4)/5 = 1/5$. Expected time $E[T_5] = 5/1 = 5$.
- Total additional time is $2.5 + 5 = 7.5$.

Answer: 7.5

3. For a set of n items, the expected time is $E[T] = nH_n$. If $n = 100$, use the approximation $H_n \approx \ln(n) + 0.577$ to estimate the expected number of draws.

- Substitute $n = 100$ into the approximation: $H_{100} \approx \ln(100) + 0.577$.
- Calculate $\ln(100) \approx 4.605$.
- Sum them: $H_{100} \approx 4.605 + 0.577 = 5.182$.
- Multiply by n : $E[T] \approx 100 \times 5.182 = 518.2$.

Answer: 518.2