

The Hypergeometric Distribution

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Practice problems

1. A box contains 15 lightbulbs, 3 of which are defective. If 4 bulbs are chosen at random without replacement, what is the probability that exactly 1 is defective?
2. In a group of 12 students, 5 are seniors and 7 are juniors. If a committee of 3 is chosen at random, what is the probability that the committee contains at least 2 seniors?
3. A deck of 20 cards has 6 red cards and 14 black cards. If 5 cards are drawn without replacement, what is the probability that exactly 3 are red?

Solutions

1. A box contains 15 lightbulbs, 3 of which are defective. If 4 bulbs are chosen at random without replacement, what is the probability that exactly 1 is defective?

- Identify parameters: $N = 15$, $K = 3$, $n = 4$, $k = 1$.
- Calculate ways to pick 1 defective: $\binom{3}{1} = 3$.
- Calculate ways to pick 3 non-defective: $\binom{12}{3} = \frac{12 \times 11 \times 10}{3 \times 2 \times 1} = 220$.
- Calculate total ways to pick 4 bulbs: $\binom{15}{4} = \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1} = 1365$.
- Compute probability: $P(X = 1) = \frac{3 \times 220}{1365} = \frac{660}{1365} \approx 0.4835$.

Answer: 0.4835

2. In a group of 12 students, 5 are seniors and 7 are juniors. If a committee of 3 is chosen at random, what is the probability that the committee contains at least 2 seniors?

- Identify parameters: $N = 12$, $K = 5$, $n = 3$. We need $P(X \geq 2) = P(X = 2) + P(X = 3)$.
- For $k = 2$: $P(X = 2) = \frac{\binom{5}{2}\binom{7}{1}}{\binom{12}{3}} = \frac{10 \times 7}{220} = \frac{70}{220}$.
- For $k = 3$: $P(X = 3) = \frac{\binom{5}{3}\binom{7}{0}}{\binom{12}{3}} = \frac{10 \times 1}{220} = \frac{10}{220}$.
- Sum the probabilities: $\frac{70}{220} + \frac{10}{220} = \frac{80}{220} \approx 0.3636$.

Answer: 0.3636

3. A deck of 20 cards has 6 red cards and 14 black cards. If 5 cards are drawn without replacement, what is the probability that exactly 3 are red?

- Identify parameters: $N = 20$, $K = 6$, $n = 5$, $k = 3$.
- Calculate ways to pick 3 red: $\binom{6}{3} = 20$.
- Calculate ways to pick 2 black: $\binom{14}{2} = \frac{14 \times 13}{2} = 91$.
- Calculate total ways to pick 5 cards: $\binom{20}{5} = 15504$.
- Compute probability: $P(X = 3) = \frac{20 \times 91}{15504} = \frac{1820}{15504} \approx 0.1174$.

Answer: 0.1174