The Poisson distribution II — as binomial's limit

Lesson 37 · Module 2 — Discrete Random Variables · Probability & Statistics · dailymaths.uk/probstat

Practice problems

1. A manufacturer finds that 0.4% of their components are defective. In a random sample of 500 components, what is the probability that exactly 2 are defective? Use the Poisson approximation.
2. A rare side effect occurs in 0.1% of patients taking a medication. In a group of 2000 patients, what is the probability that 3 or fewer patients experience the side effect? Use the Poisson approximation.
3. A lottery ticket has a 1 in $10,000$ chance of winning a small prize. If you buy $10,000$ tickets, what is the probability that you win exactly 1 prize? Compare the Poisson approximation to the exact Binomial result.

Solutions

1. A manufacturer finds that 0.4% of their components are defective. In a random sample of 500 components, what is the probability that exactly 2 are defective? Use the Poisson approximation.

   - Identify $n = 500$ and $p = 0.004$.
   - Calculate $\lambda = np = 500 \times 0.004 = 2$.
   - Use the Poisson PMF for $k = 2$: $P(X = 2) = \frac{2^2 e^{-2}}{2!}$.
   - Compute $P(X = 2) = \frac{4 \times 0.1353}{2} = 0.2707$.

   Answer: 0.2707

2. A rare side effect occurs in 0.1% of patients taking a medication. In a group of 2000 patients, what is the probability that 3 or fewer patients experience the side effect? Use the Poisson approximation.

   - Identify $n = 2000$ and $p = 0.001$.
   - Calculate $\lambda = np = 2000 \times 0.001 = 2$.
   - Find $P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$.
   - Calculate each: $P(0) = e^{-2} \approx 0.1353$, $P(1) = 2e^{-2} \approx 0.2707$, $P(2) = \frac{4e^{-2}}{2} \approx 0.2707$, $P(3) = \frac{8e^{-2}}{6} \approx 0.1804$.
   - Sum them: $0.1353 + 0.2707 + 0.2707 + 0.1804 = 0.8571$.

   Answer: 0.8571

3. A lottery ticket has a 1 in $10,000$ chance of winning a small prize. If you buy $10,000$ tickets, what is the probability that you win exactly 1 prize? Compare the Poisson approximation to the exact Binomial result.

   - For Poisson: $n = 10,000, p = 0.0001 \implies \lambda = 1$.
   - Calculate $P(X = 1) = \frac{1^1 e^{-1}}{1!} = e^{-1} \approx 0.3679$.
   - For Binomial: $P(X = 1) = \binom{10000}{1}(0.0001)^1(0.9999)^{9999}$.
   - Calculate $10000 \times 0.0001 \times (0.9999)^{9999} = 1 \times 0.36788 \approx 0.3679$.
   - The results are nearly identical.

   Answer: 0.3679

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