

The Poisson Distribution I — The Law of Rare Events

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Practice problems

1. A small town's fire department receives an average of 3.5 emergency calls per hour. What is the probability that they receive exactly 2 calls in a given hour?
2. A book has an average of 0.2 typos per page. What is the probability that a randomly selected 5-page section contains exactly 1 typo?
3. A website receives an average of 4 hits per minute. What is the probability that the website receives more than 1 hit in a given minute?

Solutions

1. A small town's fire department receives an average of 3.5 emergency calls per hour. What is the probability that they receive exactly 2 calls in a given hour?

- Identify the rate $\lambda = 3.5$ and the number of events $k = 2$.
- Apply the Poisson formula: $P(X = 2) = \frac{3.5^2 e^{-3.5}}{2!}$.
- Calculate $3.5^2 = 12.25$ and $2! = 2$.
- Compute $P(X = 2) = \frac{12.25 \times 0.0302}{2} \approx 0.1850$.

Answer: 0.1850

2. A book has an average of 0.2 typos per page. What is the probability that a randomly selected 5-page section contains exactly 1 typo?

- First, scale the rate to the 5-page interval: $\lambda = 0.2 \times 5 = 1$.
- Identify $k = 1$.
- Apply the formula: $P(X = 1) = \frac{1^1 e^{-1}}{1!}$.
- Simplify to $e^{-1} \approx 0.3679$.

Answer: 0.3679

3. A website receives an average of 4 hits per minute. What is the probability that the website receives more than 1 hit in a given minute?

- Identify $\lambda = 4$. We want $P(X > 1)$.
- Use the complement: $P(X > 1) = 1 - [P(X = 0) + P(X = 1)]$.
- Calculate $P(X = 0) = \frac{4^0 e^{-4}}{0!} = e^{-4} \approx 0.0183$.
- Calculate $P(X = 1) = \frac{4^1 e^{-4}}{1!} = 4e^{-4} \approx 0.0733$.
- Sum the probabilities: $0.0183 + 0.0733 = 0.0916$.
- Subtract from one: $1 - 0.0916 = 0.9084$.

Answer: 0.9084