

The geometric distribution — waiting for success

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Practice problems

1. A fair coin is flipped repeatedly. What is the probability that the first head appears on the 4th flip?
2. A software bug occurs randomly with a probability of 0.05 per day. What is the probability that the first bug is detected within the first 10 days?
3. An archer hits a target with probability 0.3. What is the expected number of shots they must take to hit the target for the first time, and what is the probability they take more than 5 shots?

Solutions

1. A fair coin is flipped repeatedly. What is the probability that the first head appears on the 4th flip?

- Identify the probability of success $p = 0.5$ and the trial number $k = 4$.
- Use the geometric PMF: $P(X = 4) = (1 - 0.5)^{4-1} \cdot 0.5$.
- Calculate $(0.5)^3 \cdot 0.5 = 0.125 \cdot 0.5 = 0.0625$.

Answer: 0.0625

2. A software bug occurs randomly with a probability of 0.05 per day. What is the probability that the first bug is detected within the first 10 days?

- Identify $p = 0.05$ and $k = 10$.
- Use the cumulative formula: $P(X \leq 10) = 1 - (1 - 0.05)^{10}$.
- Calculate $1 - (0.95)^{10} \approx 1 - 0.5987 = 0.4013$.

Answer: 0.4013

3. An archer hits a target with probability 0.3. What is the expected number of shots they must take to hit the target for the first time, and what is the probability they take more than 5 shots?

- The expected value is $E[X] = 1/p = 1/0.3 \approx 3.33$.
- The probability of taking more than 5 shots is the probability of failing the first 5 shots: $P(X > 5) = (1 - 0.3)^5$.
- Calculate $(0.7)^5 = 0.16807$.

Answer: $E[X] \approx 3.33$, $P(X > 5) \approx 0.1681$