

Binomial in action: quality control, polling, streaks

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Practice problems

1. A factory produces lightbulbs with a 10% defect rate. If a random sample of 15 bulbs is chosen, what is the probability that exactly 2 are defective?
2. A basketball player makes 70% of their free throws. If they take 6 shots, what is the probability they make at least 5?
3. A software company finds that 5% of its users encounter a specific bug. In a random sample of 100 users, what is the expected number of users who encounter the bug, and what is the standard deviation?

Solutions

1. A factory produces lightbulbs with a 10% defect rate. If a random sample of 15 bulbs is chosen, what is the probability that exactly 2 are defective?

- Identify parameters: $n = 15$, $p = 0.1$, $k = 2$.
- Apply the binomial formula: $P(X = 2) = \binom{15}{2}(0.1)^2(0.9)^{13}$.
- Compute the combination: $\binom{15}{2} = \frac{15 \times 14}{2} = 105$.
- Compute the powers: $(0.1)^2 = 0.01$ and $(0.9)^{13} \approx 0.2542$.
- Multiply the results: $105 \times 0.01 \times 0.2542 \approx 0.2669$.

Answer: 0.2669

2. A basketball player makes 70% of their free throws. If they take 6 shots, what is the probability they make at least 5?

- Identify parameters: $n = 6$, $p = 0.7$.
- We need $P(X \geq 5) = P(X = 5) + P(X = 6)$.
- Calculate $P(X = 5) = \binom{6}{5}(0.7)^5(0.3)^1 = 6 \times 0.16807 \times 0.3 = 0.3025$.
- Calculate $P(X = 6) = \binom{6}{6}(0.7)^6(0.3)^0 = 1 \times 0.1176 \times 1 = 0.1176$.
- Sum the probabilities: $0.3025 + 0.1176 = 0.4201$.

Answer: 0.4201

3. A software company finds that 5% of its users encounter a specific bug. In a random sample of 100 users, what is the expected number of users who encounter the bug, and what is the standard deviation?

- Identify parameters: $n = 100$, $p = 0.05$.
- Calculate expectation: $E[X] = np = 100 \times 0.05 = 5$.
- Calculate variance: $\text{Var}(X) = np(1 - p) = 100 \times 0.05 \times 0.95 = 4.75$.
- Calculate standard deviation: $\sigma = \sqrt{4.75} \approx 2.1794$.

Answer: $E[X]=5$, $\sigma \approx 2.1794$