

The Bernoulli and Binomial Distributions

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Practice problems

1. A fair six-sided die is rolled 6 times. What is the probability of rolling exactly 2 sixes?
2. A factory produces lightbulbs with a 3% defect rate. If a random sample of 20 bulbs is tested, what is the expected number of defective bulbs and the variance?
3. A multiple-choice test has 10 questions, each with 4 options. If a student guesses randomly on every question, what is the probability they get exactly 3 correct?

Solutions

1. A fair six-sided die is rolled 6 times. What is the probability of rolling exactly 2 sixes?

- Identify parameters: $n = 6$, $p = 1/6$, $k = 2$.
- Use the binomial formula: $P(X = 2) = \binom{6}{2}(1/6)^2(5/6)^{6-2}$.
- Calculate the combination: $\binom{6}{2} = 15$.
- Calculate the powers: $(1/6)^2 = 1/36$ and $(5/6)^4 = 625/1296$.
- Multiply together: $15 \cdot (1/36) \cdot (625/1296) = 9375/46656 \approx 0.2009$.

Answer: 0.2009

2. A factory produces lightbulbs with a 3% defect rate. If a random sample of 20 bulbs is tested, what is the expected number of defective bulbs and the variance?

- Identify parameters: $n = 20$, $p = 0.03$.
- Calculate mean: $E[X] = np = 20 \cdot 0.03 = 0.6$.
- Calculate variance: $\text{Var}(X) = np(1 - p) = 20 \cdot 0.03 \cdot 0.97$.
- Compute the product: $0.6 \cdot 0.97 = 0.582$.

Answer: $E[X] = 0.6$, $\text{Var}(X) = 0.582$

3. A multiple-choice test has 10 questions, each with 4 options. If a student guesses randomly on every question, what is the probability they get exactly 3 correct?

- Identify parameters: $n = 10$, $p = 1/4 = 0.25$, $k = 3$.
- Use the binomial formula: $P(X = 3) = \binom{10}{3}(0.25)^3(0.75)^7$.
- Calculate the combination: $\binom{10}{3} = 120$.
- Calculate the powers: $(0.25)^3 = 0.015625$ and $(0.75)^7 \approx 0.13348$.
- Multiply together: $120 \cdot 0.015625 \cdot 0.13348 \approx 0.2503$.

Answer: 0.2503