Variance and Standard Deviation

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Practice problems

1. A random variable X has the following probability distribution: $P(X = 1) = 0.2$, $P(X = 2) = 0.5$, and $P(X = 3) = 0.3$. Calculate the variance and standard deviation of X.
2. Using the computational shortcut $\text{Var}(X) = E[X^2] - (E[X])^2$, find the variance of a random variable X where $P(X = 0) = 0.4$, $P(X = 10) = 0.4$, and $P(X = 20) = 0.2$.
3. If a random variable X has a variance of 0, what can you conclude about the values X can take and its probability mass function?

Solutions

1. A random variable X has the following probability distribution: $P(X = 1) = 0.2$, $P(X = 2) = 0.5$, and $P(X = 3) = 0.3$. Calculate the variance and standard deviation of X.

   - First, find the expected value: $E[X] = 1(0.2) + 2(0.5) + 3(0.3) = 0.2 + 1.0 + 0.9 = 2.1$.
   - Next, find the squared deviations: $(1 - 2.1)^2 = 1.21$, $(2 - 2.1)^2 = 0.01$, $(3 - 2.1)^2 = 0.81$.
   - Weight these by probabilities: $1.21(0.2)+0.01(0.5)+0.81(0.3) = 0.242+0.005+0.243 = 0.49$.
   - The variance is 0.49.
   - The standard deviation is $\sqrt{0.49} = 0.7$.

   Answer: \textVar(X) = 0.49, \sigma = 0.7

2. Using the computational shortcut $\text{Var}(X) = E[X^2] - (E[X])^2$, find the variance of a random variable X where $P(X = 0) = 0.4$, $P(X = 10) = 0.4$, and $P(X = 20) = 0.2$.

   - Calculate $E[X] = 0(0.4) + 10(0.4) + 20(0.2) = 0 + 4 + 4 = 8$.
   - Calculate $E[X^2] = 0^2(0.4) + 10^2(0.4) + 20^2(0.2) = 0 + 40 + 80 = 120$.
   - Apply the shortcut: $\text{Var}(X) = 120 - 8^2 = 120 - 64 = 56$.

   Answer: \textVar(X) = 56

3. If a random variable X has a variance of 0, what can you conclude about the values X can take and its probability mass function?

   - The variance is defined as $\sum(x - E[X])^2 P(X = x)$.
   - Since $(x - E[X])^2$ is always non-negative and $P(X = x)$ is non-negative, the sum can only be zero if every term is zero.
   - This means for every x where $P(X = x) > 0$, we must have $x = E[X]$.
   - Therefore, X must be a constant random variable that takes only one value with probability 1.

   Answer: X is a constant; $P(X = E[X]) = 1$.
