Expectation I — the centre of mass

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Practice problems

1. A random variable X has the following probability mass function: $P(X = 10) = 0.2$, $P(X = 20) = 0.5$, and $P(X = 30) = 0.3$. Calculate $E[X]$.
2. In a game, you flip a fair coin. If it is heads, you win 5. If it is tails, you lose 2. What is the expected value of your winnings per game?
3. A discrete random variable X takes values $1, 2, 3, 4$ with probabilities $0.1, 0.3, 0.4, 0.2$ respectively. If we define a new random variable $Y = 2X + 3$, find $E[Y]$ using the values of X.

Solutions

1. A random variable X has the following probability mass function: $P(X = 10) = 0.2$, $P(X = 20) = 0.5$, and $P(X = 30) = 0.3$. Calculate $E[X]$.

    - Identify the values x_i and their probabilities p_i: $(10, 0.2), (20, 0.5), (30, 0.3)$.
    - Apply the expectation formula: $E[X] = 10(0.2) + 20(0.5) + 30(0.3)$.
    - Compute each term: $2 + 10 + 9$.
    - Sum the terms: 21.

    Answer: 21

2. In a game, you flip a fair coin. If it is heads, you win 5. If it is tails, you lose 2. What is the expected value of your winnings per game?

    - The possible outcomes are $x_1 = 5$ and $x_2 = -2$.
    - Since the coin is fair, $P(X = 5) = 0.5$ and $P(X = -2) = 0.5$.
    - Calculate the expectation: $E[X] = 5(0.5) + (-2)(0.5)$.
    - Simplify: $2.5 - 1.0 = 1.5$.

    Answer: 1.5

3. A discrete random variable X takes values $1, 2, 3, 4$ with probabilities $0.1, 0.3, 0.4, 0.2$ respectively. If we define a new random variable $Y = 2X + 3$, find $E[Y]$ using the values of X.

    - First, find $E[X]$: $E[X] = 1(0.1) + 2(0.3) + 3(0.4) + 4(0.2)$.
    - Compute $E[X]$: $0.1 + 0.6 + 1.2 + 0.8 = 2.7$.
    - Determine the values of Y for each X: Y takes values $2(1) + 3 = 5, 2(2) + 3 = 7, 2(3) + 3 = 9, 2(4) + 3 = 11$.
    - Calculate $E[Y]$ using the same probabilities: $E[Y] = 5(0.1) + 7(0.3) + 9(0.4) + 11(0.2)$.
    - Compute $E[Y]$: $0.5 + 2.1 + 3.6 + 2.2 = 8.4$.

    Answer: 8.4
