

Random variables: numbers attached to randomness

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Practice problems

1. A bag contains 3 red balls and 2 blue balls. Two balls are drawn without replacement. Let X be the number of red balls drawn. Find the probability distribution of X .
2. A fair coin is tossed 4 times. Let X be the number of heads. Calculate $P(X = 2)$.
3. A random variable X has the following distribution: $P(X = 1) = 0.2$, $P(X = 2) = 0.5$, and $P(X = 3) = k$. Find the value of k and determine $P(X < 3)$.

Solutions

1. A bag contains 3 red balls and 2 blue balls. Two balls are drawn without replacement. Let X be the number of red balls drawn. Find the probability distribution of X .

- The possible values for X are 0, 1, or 2.
- For $X = 0$, both balls must be blue: $P(X = 0) = \frac{2}{5} \times \frac{1}{4} = \frac{2}{20} = 0.1$.
- For $X = 2$, both balls must be red: $P(X = 2) = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = 0.3$.
- For $X = 1$, one red and one blue in any order: $P(X = 1) = (\frac{3}{5} \times \frac{2}{4}) + (\frac{2}{5} \times \frac{3}{4}) = \frac{6}{20} + \frac{6}{20} = \frac{12}{20} = 0.6$.
- Check sum: $0.1 + 0.6 + 0.3 = 1.0$.

Answer: $P(X=0)=0.1$, $P(X=1)=0.6$, $P(X=2)=0.3$

2. A fair coin is tossed 4 times. Let X be the number of heads. Calculate $P(X = 2)$.

- The total number of outcomes in the sample space is $2^4 = 16$.
- The number of ways to get exactly 2 heads is given by the combination formula $\binom{4}{2}$.
- $\binom{4}{2} = \frac{4 \times 3}{2 \times 1} = 6$.
- The probability is the number of successful outcomes divided by the total: $P(X = 2) = \frac{6}{16} = \frac{3}{8} = 0.375$.

Answer: 0.375

3. A random variable X has the following distribution: $P(X = 1) = 0.2$, $P(X = 2) = 0.5$, and $P(X = 3) = k$. Find the value of k and determine $P(X < 3)$.

- The sum of all probabilities must be 1: $0.2 + 0.5 + k = 1$.
- Solving for k : $0.7 + k = 1 \implies k = 0.3$.
- The event $X < 3$ includes the values $X = 1$ and $X = 2$.
- Summing these probabilities: $P(X < 3) = P(X = 1) + P(X = 2) = 0.2 + 0.5 = 0.7$.

Answer: $k=0.3$, $P(X < 3)=0.7$