Module 1 Review + Self-Test

Lesson 26 · Module 1 — Probability Foundations · Probability & Statistics · dailymaths.uk/probstat

Practice problems

1. A committee of 4 people is to be chosen from a group of 6 men and 8 women. What is the probability that the committee contains exactly 2 men and 2 women?
2. A bag contains 5 red and 5 blue balls. You draw 3 balls without replacement. What is the probability that at least one ball is red?
3. In a certain population, 1% have a rare disease. A test for the disease is 99% accurate for those who have it (sensitivity) and 95% accurate for those who do not (specificity). If a person tests positive, what is the probability they actually have the disease?

Solutions

1. A committee of 4 people is to be chosen from a group of 6 men and 8 women. What is the probability that the committee contains exactly 2 men and 2 women?

   - Calculate total ways to choose 4 from 14: $\binom{14}{4} = 1001$.
   - Calculate ways to choose 2 men from 6: $\binom{6}{2} = 15$.
   - Calculate ways to choose 2 women from 8: $\binom{8}{2} = 28$.
   - Multiply successful outcomes: $15 \times 28 = 420$.
   - Divide by total: $420/1001 \approx 0.4196$.

   Answer: 0.4196

2. A bag contains 5 red and 5 blue balls. You draw 3 balls without replacement. What is the probability that at least one ball is red?

   - Use the complement rule: $P(\text{at least one red}) = 1 - P(\text{no red})$.
   - No red means all 3 are blue.
   - Ways to choose 3 blue from 5: $\binom{5}{3} = 10$.
   - Total ways to choose 3 from 10: $\binom{10}{3} = 120$.
   - Probability of no red: $10/120 = 1/12$.
   - Subtract from one: $1 - 1/12 = 11/12 \approx 0.9167$.

   Answer: 0.9167

3. In a certain population, 1% have a rare disease. A test for the disease is 99% accurate for those who have it (sensitivity) and 95% accurate for those who do not (specificity). If a person tests positive, what is the probability they actually have the disease?

   - Prior $P(D) = 0.01$, so $P(D^c) = 0.99$.
   - Likelihoods: $P(+|D) = 0.99$ and $P(+|D^c) = 1 - 0.95 = 0.05$.
   - Total probability of positive $P(+) = (0.99)(0.01) + (0.05)(0.99) = 0.0099 + 0.0495 = 0.0594$.
   - Apply Bayes: $P(D|+) = \frac{P(+|D)P(D)}{P(+)} = \frac{0.0099}{0.0594}$.
   - Calculate result: $0.0099/0.0594 = 1/6 \approx 0.1667$.

   Answer: 0.1667
