Probability clinic — classic puzzles

Practice problems

1. A fair coin is flipped 10 times. What is the probability of getting at least 2 heads?
2. An urn contains 4 white balls and 6 black balls. Three balls are drawn without replacement. What is the probability that exactly 2 are white?
3. You have three coins: one is double-headed, one is double-tailed, and one is fair. You pick one at random and flip it; it shows heads. What is the probability it is the fair coin?

Solutions

1. A fair coin is flipped 10 times. What is the probability of getting at least 2 heads?

   - The complement of 'at least 2 heads' is '0 heads' or '1 head'.
   - The probability of 0 heads is $\binom{10}{0}(0.5)^{10} = 1/1024$.
   - The probability of 1 head is $\binom{10}{1}(0.5)^{10} = 10/1024$.
   - The probability of the complement is $11/1024$.
   - The final probability is $1 - 11/1024 = 1013/1024$.

   Answer: $1013/1024 \approx 0.9893$

2. An urn contains 4 white balls and 6 black balls. Three balls are drawn without replacement. What is the probability that exactly 2 are white?

   - The total number of ways to choose 3 balls from 10 is $\binom{10}{3} = 120$.
   - The number of ways to choose 2 white balls from 4 is $\binom{4}{2} = 6$.
   - The number of ways to choose 1 black ball from 6 is $\binom{6}{1} = 6$.
   - The number of successful outcomes is $6 \times 6 = 36$.
   - The probability is $36/120 = 3/10$.

   Answer: 0.3

3. You have three coins: one is double-headed, one is double-tailed, and one is fair. You pick one at random and flip it; it shows heads. What is the probability it is the fair coin?

   - Let F be fair, H be double-headed, T be double-tailed. $P(F) = P(H) = P(T) = 1/3$.
   - The probability of heads given fair is $P(\text{heads}|F) = 1/2$.
   - The probability of heads given double-headed is $P(\text{heads}|H) = 1$.
   - The probability of heads given double-tailed is $P(\text{heads}|T) = 0$.
   - Total probability of heads: $P(\text{heads}) = (1/3)(1/2) + (1/3)(1) + (1/3)(0) = 1/6 + 1/3 = 1/2$.
   - Using Bayes' theorem: $P(F|\text{heads}) = \frac{P(\text{heads}|F)P(F)}{P(\text{heads})} = \frac{(1/2)(1/3)}{1/2} = 1/3$.

   Answer: $1/3$
