Monty Hall — resolved forever, with simulation

Practice problems

1. In a version of the Monty Hall game, there are 5 doors. You pick one. The host, who knows where the car is, opens 2 doors to reveal goats. What is the probability of winning if you switch to one of the remaining 2 closed doors?

2. Suppose there are 3 doors. You pick door 1. The host opens door 3 and reveals a goat. However, the host did NOT know where the car was; he just happened to open a goat door by luck. What is the probability that the car is behind door 2?

3. In a 100-door Monty Hall game, you pick one door. The host opens 98 doors to reveal goats. What is the probability of winning if you switch to the last remaining door?

Solutions

1. In a version of the Monty Hall game, there are 5 doors. You pick one. The host, who knows where the car is, opens 2 doors to reveal goats. What is the probability of winning if you switch to one of the remaining 2 closed doors?

   - The initial probability of picking the car is 1/5.
   - The probability that the car is among the other 4 doors is 4/5.
   - The host eliminates 2 of those 4 doors, leaving 2 closed doors in that group.
   - The 4/5 probability is now split equally between the 2 remaining doors.
   - The probability for each remaining door is $(4/5)/2 = 2/5$.

   Answer: 2/5

2. Suppose there are 3 doors. You pick door 1. The host opens door 3 and reveals a goat. However, the host did NOT know where the car was; he just happened to open a goat door by luck. What is the probability that the car is behind door 2?

   - Let C_i be the event the car is behind door i, and G_3 be the event the host opens door 3 and finds a goat.
   - The prior probabilities are $P(C_1) = P(C_2) = P(C_3) = 1/3$.
   - The likelihoods are $P(G_3|C_1) = 1/2$ (he could pick door 2 or 3), $P(G_3|C_2) = 1/2$ (he picks from doors 2 and 3, and 3 is a goat), $P(G_3|C_3) = 0$.
   - Using Bayes: $P(C_2|G_3) = \frac{(1/2)(1/3)}{(1/2)(1/3)+(1/2)(1/3)} = 1/2$.

   Answer: 1/2

3. In a 100-door Monty Hall game, you pick one door. The host opens 98 doors to reveal goats. What is the probability of winning if you switch to the last remaining door?

   - The probability your first choice was correct is 1/100.
   - The probability the car is in the other 99 doors is 99/100.
   - The host eliminates 98 of those 99 doors.
   - The entire 99/100 probability is now concentrated in the single remaining door.
   - Switching wins if the car was in the group of 99 doors.

   Answer: 99/100
