

Bayes' theorem I — the formula, via tree diagrams

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Practice problems

1. A factory has two lines. Line 1 produces 70% of the parts with a 3% defect rate. Line 2 produces 30% of the parts with a 8% defect rate. If a part is chosen at random and found to be defective, what is the probability it came from Line 1?
2. A weather app predicts rain with 80% accuracy when it actually rains, and it predicts no rain with 90% accuracy when it does not rain. The climatological probability of rain on any given day is 20%. If the app predicts rain today, what is the probability that it actually rains?
3. Three urns contain balls. Urn A has 2 red and 3 blue. Urn B has 4 red and 1 blue. Urn C has 3 red and 2 blue. An urn is chosen at random (equal probability) and a ball is drawn. If the ball is red, what is the probability it came from Urn B?

Solutions

1. A factory has two lines. Line 1 produces 70% of the parts with a 3% defect rate. Line 2 produces 30% of the parts with a 8% defect rate. If a part is chosen at random and found to be defective, what is the probability it came from Line 1?

- Identify priors: $P(L1) = 0.7$, $P(L2) = 0.3$.
- Identify likelihoods: $P(D|L1) = 0.03$, $P(D|L2) = 0.08$.
- Calculate total probability of defect: $P(D) = (0.03 \times 0.7) + (0.08 \times 0.3) = 0.021 + 0.024 = 0.045$.
- Apply Bayes' theorem: $P(L1|D) = \frac{0.021}{0.045} = \frac{21}{45} = \frac{7}{15}$.

Answer: $7/15$ \approx 0.4667

2. A weather app predicts rain with 80% accuracy when it actually rains, and it predicts no rain with 90% accuracy when it does not rain. The climatological probability of rain on any given day is 20%. If the app predicts rain today, what is the probability that it actually rains?

- Priors: $P(Rain) = 0.2$, $P(NoRain) = 0.8$.
- Likelihoods: $P(Pred|Rain) = 0.8$, $P(Pred|NoRain) = 1 - 0.9 = 0.1$.
- Total probability of prediction: $P(Pred) = (0.8 \times 0.2) + (0.1 \times 0.8) = 0.16 + 0.08 = 0.24$.
- Posterior: $P(Rain|Pred) = \frac{0.16}{0.24} = \frac{2}{3}$.

Answer: $2/3$ \approx 0.6667

3. Three urns contain balls. Urn A has 2 red and 3 blue. Urn B has 4 red and 1 blue. Urn C has 3 red and 2 blue. An urn is chosen at random (equal probability) and a ball is drawn. If the ball is red, what is the probability it came from Urn B?

- Priors: $P(A) = P(B) = P(C) = 1/3$.
- Likelihoods: $P(R|A) = 2/5$, $P(R|B) = 4/5$, $P(R|C) = 3/5$.
- Total probability of red: $P(R) = (1/3 \times 2/5) + (1/3 \times 4/5) + (1/3 \times 3/5) = (2 + 4 + 3)/15 = 9/15 = 3/5$.
- Posterior: $P(B|R) = \frac{(1/3 \times 4/5)}{3/5} = \frac{4/15}{9/15} = 4/9$.

Answer: $4/9$ \approx 0.4444