

The Law of Total Probability

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Practice problems

1. A company has three factories: A, B, and C. Factory A produces 40% of the products, B produces 35%, and C produces 25%. The defect rates are 2% for A, 3% for B, and 5% for C. What is the probability that a randomly chosen product is defective?
2. A weather forecaster says there is a 30% chance of rain tomorrow. If it rains, the probability that a commuter is late for work is 60%. If it does not rain, the probability they are late is 10%. What is the probability the commuter is late tomorrow?
3. Two boxes contain coins. Box 1 has 3 gold and 7 silver coins. Box 2 has 8 gold and 2 silver coins. A box is chosen by flipping a fair coin, and then a coin is drawn from that box. What is the probability that the drawn coin is silver?

Solutions

1. A company has three factories: A, B, and C. Factory A produces 40% of the products, B produces 35%, and C produces 25%. The defect rates are 2% for A, 3% for B, and 5% for C. What is the probability that a randomly chosen product is defective?

- Identify the scenarios: $P(A) = 0.40$, $P(B) = 0.35$, $P(C) = 0.25$.
- Identify conditional probabilities: $P(Def|A) = 0.02$, $P(Def|B) = 0.03$, $P(Def|C) = 0.05$.
- Apply the Law of Total Probability: $P(Def) = (0.40)(0.02) + (0.35)(0.03) + (0.25)(0.05)$.
- Calculate: $P(Def) = 0.008 + 0.0105 + 0.0125 = 0.031$.

Answer: 0.031

2. A weather forecaster says there is a 30% chance of rain tomorrow. If it rains, the probability that a commuter is late for work is 60%. If it does not rain, the probability they are late is 10%. What is the probability the commuter is late tomorrow?

- Identify scenarios: $P(Rain) = 0.30$, $P(NoRain) = 0.70$.
- Identify conditional probabilities: $P(Late|Rain) = 0.60$, $P(Late|NoRain) = 0.10$.
- Apply the Law of Total Probability: $P(Late) = (0.30)(0.60) + (0.70)(0.10)$.
- Calculate: $P(Late) = 0.18 + 0.07 = 0.25$.

Answer: 0.25

3. Two boxes contain coins. Box 1 has 3 gold and 7 silver coins. Box 2 has 8 gold and 2 silver coins. A box is chosen by flipping a fair coin, and then a coin is drawn from that box. What is the probability that the drawn coin is silver?

- Identify scenarios: $P(Box1) = 0.5$, $P(Box2) = 0.5$.
- Identify conditional probabilities: $P(Silver|Box1) = 7/10 = 0.7$, $P(Silver|Box2) = 2/10 = 0.2$.
- Apply the Law of Total Probability: $P(Silver) = (0.5)(0.7) + (0.5)(0.2)$.
- Calculate: $P(Silver) = 0.35 + 0.10 = 0.45$.

Answer: 0.45