

Independence — what it really means (and doesn't)

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Practice problems

1. Given $P(A) = 0.5$, $P(B) = 0.4$, and $P(A \cap B) = 0.2$, are events A and B independent? Explain why.
2. Two events X and Y are disjoint and both have probabilities greater than zero. Are they independent? Why or why not?
3. A fair coin is flipped and a fair six-sided die is rolled. What is the probability that the coin shows tails and the die shows a number greater than 4?

Solutions

1. Given $P(A) = 0.5$, $P(B) = 0.4$, and $P(A \cap B) = 0.2$, are events A and B independent? Explain why.

- Calculate the product of the individual probabilities: $0.5 \cdot 0.4 = 0.2$.
- Compare this product to the given joint probability $P(A \cap B) = 0.2$.
- Since $0.2 = 0.2$, the condition for independence is met.

Answer: Yes, they are independent because $P(A \cap B) = P(A)P(B)$.

2. Two events X and Y are disjoint and both have probabilities greater than zero. Are they independent? Why or why not?

- Since they are disjoint, $P(X \cap Y) = 0$.
- Since $P(X) > 0$ and $P(Y) > 0$, their product $P(X)P(Y)$ must be greater than zero.
- Therefore, $P(X \cap Y) \neq P(X)P(Y)$.

Answer: No, they are dependent because the occurrence of one makes the probability of the other zero.

3. A fair coin is flipped and a fair six-sided die is rolled. What is the probability that the coin shows tails and the die shows a number greater than 4?

- Identify the events: A is tails, B is die > 4 .
- The events are independent because the coin does not affect the die.
- Compute $P(A) = 1/2$.
- Compute $P(B) = 2/6 = 1/3$ (outcomes 5 and 6).
- Multiply the probabilities: $1/2 \cdot 1/3 = 1/6$.

Answer: $1/6$