

Conditional probability II — multiplication rule; sampling without replacement

Lesson 19 · Module 1 — Probability Foundations · Probability & Statistics · dailymaths.uk/probstat

Practice problems

1. A standard deck of 52 cards contains 13 hearts. If three cards are drawn without replacement, what is the probability that all three are hearts?
2. An urn contains 7 white balls and 3 black balls. Two balls are drawn without replacement. What is the probability that the first is white and the second is black?
3. A box contains 5 red, 4 green, and 3 blue pens. If three pens are drawn without replacement, what is the probability that they are all different colors?

Solutions

1. A standard deck of 52 cards contains 13 hearts. If three cards are drawn without replacement, what is the probability that all three are hearts?

- The probability the first card is a heart is $13/52$.
- Given the first is a heart, there are 12 hearts left out of 51 cards, so the second is $12/51$.
- Given the first two are hearts, there are 11 hearts left out of 50 cards, so the third is $11/50$.
- Multiply these: $(13/52) \cdot (12/51) \cdot (11/50) = (1/4) \cdot (4/17) \cdot (11/50) = 11/850$.

Answer: $11/850$

2. An urn contains 7 white balls and 3 black balls. Two balls are drawn without replacement. What is the probability that the first is white and the second is black?

- The probability the first ball is white is $7/10$.
- Given the first was white, there are still 3 black balls, but only 9 balls total. The probability the second is black is $3/9$.
- Multiply the probabilities: $(7/10) \cdot (3/9) = 21/90 = 7/30$.

Answer: $7/30$

3. A box contains 5 red, 4 green, and 3 blue pens. If three pens are drawn without replacement, what is the probability that they are all different colors?

- There are $3! = 6$ different orders to draw one of each color (RGB, RBG, GRB, GBR, BRG, BGR).
- The probability of any one specific order, say Red then Green then Blue, is $(5/12) \cdot (4/11) \cdot (3/10) = 60/1320 = 1/22$.
- Since all 6 orders have the same probability, the total probability is $6 \cdot (1/22) = 6/22 = 3/11$.

Answer: $3/11$