

Conditional probability I — definition and intuition

Lesson 18 · Module 1 — Probability Foundations · Probability & Statistics · dailymaths.uk/probstat

Practice problems

1. A week has 7 days. Let B be the event that a randomly chosen day is a weekday (Monday through Friday). Let A be the event that the day starts with the letter 'T'. Find $P(A|B)$.
2. In a class of 30 students, 15 like Math, 10 like Science, and 7 like both. If a student is chosen at random and they like Science, what is the probability they also like Math?
3. Two fair coins are flipped. Given that at least one coin is heads, what is the probability that both coins are heads?

Solutions

1. A week has 7 days. Let B be the event that a randomly chosen day is a weekday (Monday through Friday). Let A be the event that the day starts with the letter 'T'. Find $P(A|B)$.
 - Identify the restricted sample space B : Monday, Tuesday, Wednesday, Thursday, Friday. The size $|B| = 5$.
 - Identify the intersection $A \cap B$: Days that are weekdays AND start with 'T'. These are Tuesday, Thursday. The size $|A \cap B| = 2$.
 - Apply the formula $P(A|B) = \frac{|A \cap B|}{|B|} = \frac{2}{5}$.

Answer: $2/5$

2. In a class of 30 students, 15 like Math, 10 like Science, and 7 like both. If a student is chosen at random and they like Science, what is the probability they also like Math?
 - Let M be liking Math and S be liking Science.
 - The condition is liking Science, so $P(S) = \frac{10}{30}$.
 - The intersection is liking both, so $P(M \cap S) = \frac{7}{30}$.
 - Calculate $P(M|S) = \frac{7/30}{10/30} = \frac{7}{10}$.

Answer: $7/10$

3. Two fair coins are flipped. Given that at least one coin is heads, what is the probability that both coins are heads?
 - The sample space is $S = \{HH, HT, TH, TT\}$.
 - The condition B is 'at least one head': $B = \{HH, HT, TH\}$. Thus $|B| = 3$.
 - The event A is 'both heads': $A = \{HH\}$.
 - The intersection $A \cap B$ is just $\{HH\}$, so $|A \cap B| = 1$.
 - The conditional probability is $P(A|B) = \frac{1}{3}$.

Answer: $1/3$