

The complement trick and 'at least one' problems

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Practice problems

1. A fair coin is flipped 5 times. What is the probability of getting at least one head?
2. A basketball player has a 70% free-throw success rate. If they take 3 shots, what is the probability they make at least one?
3. You have three independent security alarms. The first has a 90% chance of detecting an intruder, the second 80%, and the third 70%. What is the probability that at least one alarm goes off?

Solutions

1. A fair coin is flipped 5 times. What is the probability of getting at least one head?

- The probability of success (heads) is $p = 0.5$.
- The probability of failure (tails) is $1 - p = 0.5$.
- The probability of zero heads in 5 flips is $(0.5)^5 = 0.03125$.
- The probability of at least one head is $1 - 0.03125 = 0.96875$.

Answer: 0.96875

2. A basketball player has a 70% free-throw success rate. If they take 3 shots, what is the probability they make at least one?

- The probability of success is $p = 0.7$.
- The probability of failure (missing) is $1 - p = 0.3$.
- The probability of missing all 3 shots is $(0.3)^3 = 0.027$.
- The probability of making at least one is $1 - 0.027 = 0.973$.

Answer: 0.973

3. You have three independent security alarms. The first has a 90% chance of detecting an intruder, the second 80%, and the third 70%. What is the probability that at least one alarm goes off?

- Find the probability that each alarm fails: $P(F_1) = 0.1$, $P(F_2) = 0.2$, $P(F_3) = 0.3$.
- The probability that all three fail is $0.1 \times 0.2 \times 0.3 = 0.006$.
- The probability that at least one succeeds is $1 - 0.006 = 0.994$.

Answer: 0.994