The Birthday Problem

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Practice problems

1. In a group of $n = 3$ people, what is the probability that at least two people share a birthday? Assume a 365-day year.
2. If a group has $n = 30$ people, is the probability of a shared birthday greater than or less than 70%?
3. In a fictional world where a year has only 100 days, how many people n must be in a room for the probability of a shared birthday to exceed 50%?

Solutions

1. In a group of $n = 3$ people, what is the probability that at least two people share a birthday? Assume a 365-day year.

   - Find the probability that all 3 people have different birthdays: $P(\text{no match}) = \frac{365}{365} \times \frac{364}{365} \times \frac{363}{365}$.
   - Calculate the product: $P(\text{no match}) = \frac{132132}{133225} \approx 0.9918$.
   - Subtract from 1: $P(\text{match}) = 1 - 0.9918 = 0.0082$.

   Answer: 0.0082

2. If a group has $n = 30$ people, is the probability of a shared birthday greater than or less than 70%?

   - Calculate $P(\text{no match}) = \frac{P(365,30)}{365^{30}}$.
   - Using the formula, $P(\text{no match}) \approx 0.2937$.
   - Find $P(\text{match}) = 1 - 0.2937 = 0.7063$.
   - Since $0.7063 > 0.70$, the probability is greater than 70%.

   Answer: Greater than 70%

3. In a fictional world where a year has only 100 days, how many people n must be in a room for the probability of a shared birthday to exceed 50%?

   - We need $P(\text{no match}) < 0.5$, where $P(\text{no match}) = \frac{P(100,n)}{100^n}$.
   - For $n = 12$: $P(\text{no match}) = \frac{100 \times 99 \times \cdots \times 89}{100^{12}} \approx 0.501$.
   - For $n = 13$: $P(\text{no match}) = \frac{100 \times 99 \times \cdots \times 88}{100^{13}} \approx 0.441$.
   - The probability first exceeds 0.5 when $n = 13$.

   Answer: 13
