Equally likely outcomes; counting meets probability

Practice problems

1. A standard deck of 52 cards is used. If 5 cards are drawn at random without replacement, what is the probability that all 5 cards are of the same suit (a flush)?

2. A bag contains 7 red marbles and 5 blue marbles. If 3 marbles are drawn at random without replacement, what is the probability that exactly 2 are red?

3. Four people are randomly seated in a row of four chairs. What is the probability that two specific people, Alice and Bob, are NOT sitting next to each other?

Solutions

1. A standard deck of 52 cards is used. If 5 cards are drawn at random without replacement, what is the probability that all 5 cards are of the same suit (a flush)?

   - The total number of ways to choose 5 cards from 52 is $\binom{52}{5} = 2598960$.
   - There are 4 suits. For each suit, there are 13 cards, and we choose 5 of them: $\binom{13}{5} = 1287$.
   - The total number of favorable outcomes is $4 \times 1287 = 5148$.
   - The probability is $\frac{5148}{2598960} \approx 0.00198$.

   Answer: \frac51482598960 \approx 0.00198

2. A bag contains 7 red marbles and 5 blue marbles. If 3 marbles are drawn at random without replacement, what is the probability that exactly 2 are red?

   - The total number of ways to choose 3 marbles from 12 is $\binom{12}{3} = 220$.
   - The number of ways to choose 2 red marbles from 7 is $\binom{7}{2} = 21$.
   - The number of ways to choose 1 blue marble from 5 is $\binom{5}{1} = 5$.
   - The number of favorable outcomes is $21 \times 5 = 105$.
   - The probability is $\frac{105}{220} = \frac{21}{44}$.

   Answer: \frac2144

3. Four people are randomly seated in a row of four chairs. What is the probability that two specific people, Alice and Bob, are NOT sitting next to each other?

   - The total number of arrangements is $4! = 24$.
   - First, find the probability that they ARE sitting next to each other. Treat Alice and Bob as one block. There are 3! ways to arrange the block and the other two people, and 2! ways to arrange Alice and Bob within the block: $3! \times 2! = 6 \times 2 = 12$.
   - The probability they are adjacent is $\frac{12}{24} = \frac{1}{2}$.
   - The probability they are NOT adjacent is $1 - \frac{1}{2} = \frac{1}{2}$.

   Answer: \frac12
