

The axioms; probability as long-run frequency

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Practice problems

1. A sample space has four outcomes: A, B, C , and D . A probability assignment is given as $P(A) = 0.4, P(B) = 0.3, P(C) = 0.2$, and $P(D) = 0.2$. Is this a valid probability assignment? Explain why or why not.
2. In a game, the probability of winning is 0.15 and the probability of drawing is 0.30. Assuming these are the only outcomes other than losing, what is the probability of losing?
3. A biased spinner has three sections: Red, Blue, and Green. The probability of Red is $1/3$. The probability of Blue is twice the probability of Green. Find the probability of Green.

Solutions

1. A sample space has four outcomes: A, B, C , and D . A probability assignment is given as $P(A) = 0.4, P(B) = 0.3, P(C) = 0.2$, and $P(D) = 0.2$. Is this a valid probability assignment? Explain why or why not.

- Check Axiom 1: All values $0.4, 0.3, 0.2, 0.2$ are ≥ 0 .
- Check Axiom 2: Sum the probabilities: $0.4 + 0.3 + 0.2 + 0.2 = 1.1$.
- Since the sum is 1.1 , which is not equal to 1 , the normalization axiom is violated.

Answer: No, it is invalid because the sum of probabilities is 1.1 , violating $P(S) = 1$.

2. In a game, the probability of winning is 0.15 and the probability of drawing is 0.30 . Assuming these are the only outcomes other than losing, what is the probability of losing?

- Let W be winning, D be drawing, and L be losing.
- The sample space is $S = \{W, D, L\}$.
- By Axiom 2, $P(W) + P(D) + P(L) = 1$.
- Substitute the known values: $0.15 + 0.30 + P(L) = 1$.
- Solve for $P(L)$: $P(L) = 1 - 0.45 = 0.55$.

Answer: 0.55

3. A biased spinner has three sections: Red, Blue, and Green. The probability of Red is $1/3$. The probability of Blue is twice the probability of Green. Find the probability of Green.

- Let $P(G) = x$. Then $P(B) = 2x$.
- We know $P(R) = 1/3$.
- By Axiom 2, $P(R) + P(B) + P(G) = 1$.
- Substitute: $1/3 + 2x + x = 1$.
- Combine terms: $3x = 1 - 1/3 = 2/3$.
- Divide by 3 : $x = 2/9$.

Answer: $2/9$