Module 0 review + self-test

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Practice problems

1. How many ways can a committee of 4 people be chosen from a group of 10?
2. How many ways can 7 identical apples be distributed among 4 children such that some children may receive no apples?
3. A group of 5 people leave their umbrellas in a stand. If they each pick one up at random, how many ways can they do so such that exactly 2 people get their own umbrella back?

Solutions

1. How many ways can a committee of 4 people be chosen from a group of 10?

   - The order of people in a committee does not matter, so we use combinations.
   - We apply the formula $\binom{10}{4}$.
   - Calculation: $\frac{10\times9\times8\times7}{4\times3\times2\times1} = 210$.

   Answer: 210

2. How many ways can 7 identical apples be distributed among 4 children such that some children may receive no apples?

   - The apples are identical and the children are distinct, so we use stars and bars.
   - Here $n = 7$ (apples) and $k = 4$ (children).
   - The formula is $\binom{n+k-1}{k-1} = \binom{7+4-1}{4-1} = \binom{10}{3}$.
   - Calculation: $\frac{10\times9\times8}{3\times2\times1} = 120$.

   Answer: 120

3. A group of 5 people leave their umbrellas in a stand. If they each pick one up at random, how many ways can they do so such that exactly 2 people get their own umbrella back?

   - First, we choose which 2 people get their own umbrellas: $\binom{5}{2} = 10$ ways.
   - The remaining 3 people must all get the wrong umbrella, which is a derangement of 3 items: $D(3)$.
   - Calculate $D(3) = 3!(1 - 1 + 1/2 - 1/6) = 6(1/3) = 2$.
   - Total ways $= \binom{5}{2} \times D(3) = 10 \times 2 = 20$.

   Answer: 20
