

Counting gauntlet — mixed practice

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Practice problems

1. A committee of 4 people is to be chosen from a group of 10. How many different committees are possible?
2. How many ways can 7 identical apples be distributed among 3 children such that each child receives at least one apple?
3. A set of 4 letters A, B, C, D is permuted. How many of these permutations are derangements?

Solutions

1. A committee of 4 people is to be chosen from a group of 10. How many different committees are possible?

- The order of people in a committee does not matter, so we use combinations.
- We need to calculate 10 choose 4.
- $\binom{10}{4} = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$.

Answer: 210

2. How many ways can 7 identical apples be distributed among 3 children such that each child receives at least one apple?

- Since the apples are identical, we use stars and bars.
- Each child must get at least one, so we pre-allocate 3 apples.
- Remaining apples $n = 7 - 3 = 4$. Number of children $k = 3$.
- The number of ways is $\binom{4+3-1}{3-1} = \binom{6}{2} = 15$.

Answer: 15

3. A set of 4 letters A, B, C, D is permuted. How many of these permutations are derangements?

- A derangement means no letter is in its original position.
- We use the formula $D(n)$ for $n = 4$.
- $D(4) = 4!(1/0! - 1/1! + 1/2! - 1/3! + 1/4!)$.
- $D(4) = 24(1 - 1 + 1/2 - 1/6 + 1/24) = 12 - 4 + 1 = 9$.

Answer: 9