

Inclusion-exclusion I — two and three sets

Lesson 9 · Module 0 — Counting · Probability & Statistics · dailymaths.uk/probstat

Practice problems

1. In a group of students, 40 study Biology and 30 study Chemistry. 12 study both. How many students study at least one of these subjects?
2. A survey finds that 50 people like Apples, 40 like Bananas, and 30 like Cherries. 15 like Apples and Bananas, 10 like Apples and Cherries, and 12 like Bananas and Cherries. 5 people like all three. How many people like at least one of the fruits?
3. The union of two sets A and B has 100 elements. Set A has 70 elements and set B has 50 elements. How many elements are in the intersection of A and B ?

Solutions

1. In a group of students, 40 study Biology and 30 study Chemistry. 12 study both. How many students study at least one of these subjects?

- Identify the sets: $|A| = 40$ and $|B| = 30$.
- Identify the intersection: $|A \cap B| = 12$.
- Apply the formula $|A \cup B| = |A| + |B| - |A \cap B|$.
- Calculate $40 + 30 - 12 = 58$.

Answer: 58

2. A survey finds that 50 people like Apples, 40 like Bananas, and 30 like Cherries. 15 like Apples and Bananas, 10 like Apples and Cherries, and 12 like Bananas and Cherries. 5 people like all three. How many people like at least one of the fruits?

- Sum the individual sets: $50 + 40 + 30 = 120$.
- Subtract the pairwise intersections: $120 - (15 + 10 + 12) = 120 - 37 = 83$.
- Add back the triple intersection: $83 + 5 = 88$.

Answer: 88

3. The union of two sets A and B has 100 elements. Set A has 70 elements and set B has 50 elements. How many elements are in the intersection of A and B ?

- Use the formula $|A \cup B| = |A| + |B| - |A \cap B|$.
- Substitute the known values: $100 = 70 + 50 - |A \cap B|$.
- Simplify the right side: $100 = 120 - |A \cap B|$.
- Solve for $|A \cap B|$: $|A \cap B| = 120 - 100 = 20$.

Answer: 20