

Counting with repetition; stars and bars

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Practice problems

1. How many ways can 10 identical gold coins be distributed among 4 distinct pirates?
2. Find the number of positive integer solutions to the equation $x + y + z + w = 20$.
3. How many ways can 15 identical marbles be placed into 3 distinct jars such that the first jar has at least 2 marbles and the second jar has at least 3 marbles?

Solutions

1. How many ways can 10 identical gold coins be distributed among 4 distinct pirates?

- Identify $n = 10$ (coins) and $k = 4$ (pirates).
- Use the stars and bars formula for non-negative integers: $\binom{n+k-1}{k-1}$.
- Substitute the values: $\binom{10+4-1}{4-1} = \binom{13}{3}$.
- Calculate $\frac{13 \times 12 \times 11}{3 \times 2 \times 1} = 13 \times 2 \times 11 = 286$.

Answer: 286

2. Find the number of positive integer solutions to the equation $x + y + z + w = 20$.

- Positive integer solutions mean each variable must be at least 1.
- Use the 'no zeros' formula: $\binom{n-1}{k-1}$.
- Here $n = 20$ and $k = 4$.
- Calculate $\binom{20-1}{4-1} = \binom{19}{3} = \frac{19 \times 18 \times 17}{3 \times 2 \times 1} = 19 \times 3 \times 17 = 969$.

Answer: 969

3. How many ways can 15 identical marbles be placed into 3 distinct jars such that the first jar has at least 2 marbles and the second jar has at least 3 marbles?

- Pre-allocate 2 marbles to jar 1 and 3 marbles to jar 2.
- Remaining marbles: $15 - 2 - 3 = 10$.
- Now distribute 10 identical marbles into 3 distinct jars with no further restrictions.
- Use $\binom{n+k-1}{k-1}$ with $n = 10$ and $k = 3$.
- Calculate $\binom{10+3-1}{3-1} = \binom{12}{2} = \frac{12 \times 11}{2} = 66$.

Answer: 66