The Binomial Theorem

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Practice problems

1. Expand the expression $(x + 4)^3$.
2. Find the coefficient of the x^2 term in the expansion of $(2x - 1)^5$.
3. Expand $(3x + 2y)^4$.

Solutions

1. Expand the expression $(x + 4)^3$.

   - Identify $a = x$, $b = 4$, and $n = 3$.
   - Use the binomial coefficients for $n = 3$: $1, 3, 3, 1$.
   - Write the terms: $1(x^3)(4^0) + 3(x^2)(4^1) + 3(x^1)(4^2) + 1(x^0)(4^3)$.
   - Simplify the constants: $1 \cdot 1 = 1$, $3 \cdot 4 = 12$, $3 \cdot 16 = 48$, $1 \cdot 64 = 64$.

   Answer: $x^3 + 12x^2 + 48x + 64$

2. Find the coefficient of the x^2 term in the expansion of $(2x - 1)^5$.

   - Identify $a = 2x$, $b = -1$, and $n = 5$.
   - The x^2 term occurs when the power of a is 2, so $n - k = 2$, which means $k = 3$.
   - Apply the formula for the term: $\binom{5}{3}(2x)^2(-1)^3$.
   - Calculate the parts: $\binom{5}{3} = 10$, $(2x)^2 = 4x^2$, and $(-1)^3 = -1$.
   - Multiply together: $10 \cdot 4x^2 \cdot (-1) = -40x^2$.

   Answer: -40

3. Expand $(3x + 2y)^4$.

   - Identify $a = 3x$, $b = 2y$, and $n = 4$.
   - Use coefficients $1, 4, 6, 4, 1$.
   - Term 1: $1(3x)^4(2y)^0 = 81x^4$.
   - Term 2: $4(3x)^3(2y)^1 = 4(27x^3)(2y) = 216x^3y$.
   - Term 3: $6(3x)^2(2y)^2 = 6(9x^2)(4y^2) = 216x^2y^2$.
   - Term 4: $4(3x)^1(2y)^3 = 4(3x)(8y^3) = 96xy^3$.
   - Term 5: $1(3x)^0(2y)^4 = 16y^4$.

   Answer: $81x^4 + 216x^3y + 216x^2y^2 + 96xy^3 + 16y^4$
