

Combinations; the binomial coefficient

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Practice problems

1. A teacher wants to choose a group of 4 students from a class of 12 to help with a project. How many different groups can be formed?
2. A lottery requires a player to choose 6 numbers from a pool of 49. How many unique tickets are possible?
3. A committee of 5 people is to be formed from a group of 7 men and 8 women. How many ways can the committee be formed if it must contain exactly 3 women and 2 men?

Solutions

1. A teacher wants to choose a group of 4 students from a class of 12 to help with a project. How many different groups can be formed?

- Identify that order does not matter, so we use combinations with $n = 12$ and $k = 4$.
- Apply the formula: $\binom{12}{4} = \frac{12!}{4!(12-4)!} = \frac{12!}{4!8!}$.
- Simplify the fraction: $\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}$.
- Calculate the numerator: 11880.
- Calculate the denominator: 24.
- Divide: $11880/24 = 495$.

Answer: 495

2. A lottery requires a player to choose 6 numbers from a pool of 49. How many unique tickets are possible?

- The order of numbers on a ticket does not matter, so we use $\binom{49}{6}$.
- Set up the formula: $\frac{49!}{6!(49-6)!} = \frac{49!}{6!43!}$.
- Expand the numerator: $49 \times 48 \times 47 \times 46 \times 45 \times 44$.
- Divide by $6! = 720$.
- The calculation results in 13,983,816.

Answer: 13,983,816

3. A committee of 5 people is to be formed from a group of 7 men and 8 women. How many ways can the committee be formed if it must contain exactly 3 women and 2 men?

- First, calculate the ways to choose 3 women from 8: $\binom{8}{3} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$.
- Next, calculate the ways to choose 2 men from 7: $\binom{7}{2} = \frac{7 \times 6}{2 \times 1} = 21$.
- Use the multiplication principle to combine these independent choices.
- Total ways = $56 \times 21 = 1176$.

Answer: 1176